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The University of Texas at Austin

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The Developing Economist, An Undergraduate Journal of Economics at The University of Texas at Austin.

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Lastly, we would like to thank the College of Liberal Arts and Omicron Delta Epsilon for providing significant funding for the journal and for their continued support.

Thank You,
Editorial Team

The Developing Economist

An Undergraduate Journal of Economics

The editorial team is excited to publish this second volume of *The Developing Economist*. This year was highly competitive, and during the submission process we received over thirty submissions to the journal. The four papers that we selected for publication reflect outstanding undergraduate research completed by economics students from around the country.

Throughout the year, *The Developing Economist* editorial team has sought to support undergraduate research through other venues as well. Our team regularly sends a representative to the Research Student Advisory Council (RSAC) to discuss methods of improving undergraduate research on campus with students from other research-focused organizations. During these meetings we also advise the Office of Undergraduate Research and learn about new research initiatives on campus. Additionally, this year several editors attended the Federal Reserve Bank of Dallas' Economics Scholars Program conference to support fellow researchers and to raise awareness about the journal.

The editorial team is encouraged by the interest and support of students and faculty alike in our endeavor. We hope to continue promoting research at the undergraduate level, and that the papers published in *The Developing Economist* help to inspire future researchers for years to come.

Editorial Team
The Developing Economist

A Note From the Department Chair

Dear Readers:
The University of Texas Economics faculty has been thrilled with the introduction of *The Developing Economist*. We were impressed by the initiative taken by our Omicron Delta Epsilon chapter to create this undergraduate research journal. A faculty advisor did not urge the society's members to create this journal, but rather the students saw a need for an outlet where undergraduates could publish their original economics research papers. Now in its second year of existence, the journal is completely managed and edited by our undergraduates. We are proud in knowing that *The Developing Economist* is one of only a handful of undergraduate economics research journals in the entire country.

Each one of my colleagues is passionate about economics research - that's why we entered academia. We know that when a student undertakes original research, they shift from being *consumers* of knowledge (in the classroom) to *producers* of knowledge (outside the classroom). Opportunities for independent research can be limited at the undergraduate level, especially at a large University like our own. We urge students to pursue such research through an honors thesis if possible. And we promote and celebrate that research through our own honors symposia on campus. This journal provides an important outlet for this type of research, both for students at UT-Austin and other students across the country. You will no doubt be impressed by the creativity and depth of the research topics displayed within this issue.

We again applaud our students for starting this journal, and we look forward to the involvement of future students in the publication of *The Developing Economist* for many years to come!

Dr. Jason Abrevaya
Chair, Department of Economics
The University of Texas at Austin

Staff

Contents

Value-Added Real Effective Exchange Rates: Testing for Countries with High and Low Vertical Specialization in Trade

Peter A. Kallis [1]

Abstract

I test a modified value-added real effective exchange rate based on the construction by Bems and Johnson (2012) for suitability as a replacement for conventionally-constructed real effective exchange rates for countries with high vertical specialization. To do so, I construct an error-correction model using the exports of two countries with different levels of vertical specialization: Belgium and Germany. I find an insignificant relationship in the short run, but observe that in the long run, the value-added real effective exchange rate may perform better as an indicator of export competitiveness for countries with high vertical specialization than the conventional real effective exchange rate. Further analysis of the short-run relationship using an ARDL model and panel regression provides contradictory results.

I. Introduction

Over the last two decades, a growing body of theoretical and empirical literature has concerned itself with the changing landscape of the international trade market. As globalization and trade liberalization have progressed, several papers have drawn attention to and analyzed the phenomenon of vertical specialization, or the use of intermediate imports as inputs to export production.[2]

Yi (2003) argues that vertical specialization has been a major explanatory force behind the growth in world trade since 1960.[3] Similarly, Hummels, Ishii, and Yi (2001) provide evi-

[1]Special thanks to my faculty advisor, Dr. Iqbal Zaidi, and my graduate student advisor, Olivier Darmouni, for their invaluable guidance during this project.

[2]Hummels, Ishii, and Yi 2001, 76.

[3]Yi 2003, 91.

dence for a trend of steady growth in the vertical specialization share of exports between 1970 and 1990, finding that by 1990, the vertical specialization share of total exports across the countries studied accounted for 21 percent of total exports and 30 percent of total export growth.[4] More recently, Bems, Johnson, and Yi (2011) found that vertical specialization was an important factor in the decline in global trade between 2008 and 2009, accounting for 32 percent of the drop in total trade over the period.[5]

In light of this work, the role of vertical specialization as an increasingly-important driving force behind trade growth and contraction cannot be ignored. This is especially true when considering that changes in the nature of export production could have considerable ramifications on the underlying assumptions of conventional models of competitiveness.[6] Bems and Johnson (2012) consider the consequences of the emergence of vertical specialization on the theoretical framework used to construct conventional real effective exchange rates (REERs). They argue that increasing shares of vertical specialization in trade make standard REERs constructed using the Armington framework less appropriate.[7] They propose a new method of constructing REERs to reflect the growth of vertical specialization, which they name the value-added real effective exchange rate.[8] The authors demonstrate that important differences exist between the value-added real effective exchange rate (VAREER) and conventional measures of the real effective exchange rate, and suggest that increases in vertical specialization may mean that the VAREER may function as a better explanatory variable for trade fluctuations than conventional rates for a given country.[9] This paper will test for the existence of a statistically-significant difference between these two exchange rates in an effort to prove the theoretical

[4]Hummels, Ishii, and Yi 2001, 77.
[5]Bems, Johnson, and Yi 2011, 316-317.
[6]Bems and Johnson 2012, 2.
[7]Bems and Johnson 2012, 2.
[8]Bems and Johnson 2012, 3.
[9]Bems and Johnson 2012, 1-2.

prediction of Bems and Johnson (2012).

As a first step toward empirically testing the theoretical work of Bems and Johnson (2012), this paper explores the following question: does a real effective exchange rate constructed to reflect value-added prices explain the variation in a country's exports better than conventional real effective exchange rates, if that country has a high vertical specialization share of exports? To answer this question, I test two broad hypotheses drawn from the theoretical arguments made by Bems and Johnson (2012): (1) that for a given country with high vertical specialization in trade, REERs constructed using value-added prices will explain changes in exports better than conventionally-constructed REERs, and (2) that REERs constructed using value-added prices will explain fluctuations in exports better for countries with high vertical specialization than they do for countries with low vertical specialization in trade. The first hypothesis addresses the suitability of the VA-REER as a replacement for conventionally-constructed rates, and the second hypothesis addresses whether increased vertical specialization explains this suitability. Using evidence from papers on vertical specialization by Hummels, Ishii, and Yi (2001) and Breda, Cappariello, and Zizza (2008), I test these hypotheses using Belgium as a representative country with high vertical specialization and Germany as a representative country with low vertical specialization.[10]

II. Literature Review

Vertical Specialization

Hummels, Ishii, and Yi (2001) define vertical specialization as the use of "imported intermediate goods...by a country to make goods or goods-in-process which are themselves exported to another country."[11] They analyze vertical specialization between 1970 and 1990 for 14 countries and find that vertical

[10]Hummels, Ishii, and Yi 2001, 84; Breda, Cappariello, and Zizza 2008, Tab. 1-2.

[11]Hummels, Ishii, and Yi 2001, 77.

specialization as a share of total exports steadily rose for all of the OECD countries over the period examined.[12] They also observe that the vertical specialization share of exports tended to be much higher for the smallest countries in the sample and lower for the largest countries.[13]

Expanding on the analysis done by Hummels, Ishii, and Yi (2001), Breda, Cappariello, and Zizza (2008) measure and compare the import content of exports for several European countries in 1995 and 2000. They find that after controlling for energy imports, Belgium had the highest vertical specialization share of exports in 1995 and 2000 in the sample, at 39.8 percent and 44.1 percent respectively, while Germany had the second lowest vertical specialization share of exports in 1995 and the third lowest in 2000, at 20.3 percent and 26.2 percent.[14]

Real Effective Exchange Rates

Bems and Johnson (2012) modify the Armingtom framework for constructing REERs to reflect trade in value added rather than in goods wholly produced within the exporting country.[15] They base this revised framework on the claim that increased vertical specialization in world trade has changed the nature of trade competition between countries. Rather than competing against each other's similar goods on the world market, they now compete against each other's potential to add value to the supply chain.[16]

To reflect their theoretical revision, they devise a new method for calculating a country's REER by modifying both the price and trade-weight components.[17] They refer to it as the value-

[12]Hummels, Ishii, and Yi 2001, 83-5. Recognizing that changes in the price of imported oil could change their measure of each country's level of vertical specialization, the authors found it necessary to calculate the vertical specialization share of exports twice for each country and excluded energy trade in the second calculation.

[13]Hummels, Ishii, and Yi 2001, 83.

[14]Breda, Cappariello, and Zizza 2008, Tab. 2.

[15]Bems and Johnson 2012, 2-3.

[16]Bems and Johnson 2012, 2.

[17]Bems and Johnson 2012, 18.

added real effective exchange rate and present it as an alternative to the conventional REERs used by the ECB and IMF, among others.[18] The method they use to construct the VAREER introduces two changes to the conventional technique: first, they construct new bilateral trade weights that reflect value-added trade rather than total trade, and second, they replace consumer prices with GDP deflator to better reflect the value-added component of trade competitiveness.[19]

As a next step, Bems and Johnson (2012) construct annual VAREERs for 42 countries between 1970 and 2009 and compare these values with each country's conventional REER over the same period.[20] They conclude that there are important differences between the two measures of competitiveness, primarily due to the use of GDP deflator in place of consumer prices, rather than their revised construction of the bilateral trade weights.[21]

One problem with the VAREER as calculated by Bems and Johnson (2012) is that the data only exist to construct annual value-added bilateral weights between 1970 and 2009, since the authors rely heavily on annual input-output tables. The limited number of observations resulting from this technique does not provide a sufficient number of observations for reliable econometric results.[22] As a result, Bems and Johnson are not able to test their hypothesis using regression analysis.

However, given the authors' findings on the greater significance of the price-component of their real effective exchange rate, it is possible to construct a modified VAREER that uses the conventional trade weights as constructed by Bayoumi, Lee, and Jayanthi (2006) but adds in GDP deflator as a proxy for value-added prices.[23] This approach keeps true to the theoretical assertions of Bems and Johnson (2012), while sufficiently modifying their measure to rigorously test it against

[18]Bems and Johnson 2012, 15.
[19]Bems and Johnson 2012, 18.
[20]Bems and Johnson 2012, 17-24
[21]Bems and Johnson 2012, 24.
[22]Bems and Johnson 2012, 17.
[23]Bems and Johnson 2012, 24.

conventional real effective exchange rates.

Bayoumi, Lee, and Jayanthi (2006) develop the methodology that is currently used by the IMF for calculating the real effective exchange rate.[24] However, unlike Bayoumi, Lee, and Jayanthi (2006), who calculate trade weights based on a three year period (1999-2001) and apply those weights to their entire sample, I update the bilateral trade weights yearly to increase the accuracy of my results.[25]

III. Methodology

Country Selection

This paper's analysis of the VAREER is conducted as a comparison of data from two countries: Belgium and Germany. These two countries were identified as a satisfactory pair for two main reasons. First, they differ significantly in terms of the share of their exports that is explained by vertical specialization. Both Hummels, Ishii, and Yi (2001) and Breda, Cappariello, and Zizza (2008) identify Germany as a nation with relatively low vertical specialization in trade.[26] By contrast, Belgium is a prime example of a nation with high vertical specialization in trade. Breda, Cappariello, and Zizza (2008) identify it as the nation with by-far the highest level of vertical specialization among the major European countries they examine.[27]

Second, the two countries are sufficiently homogenous, save for differences in economic size and vertical specialization, thus making it less likely that any observed differences in the regression will be driven by omitted variables. Geographic affects are minimized by the selection, as the two share a common border. Both countries possess federal governments, are members of the OECD, European Union, and the Eurozone, and share

[24] Bems and Johnson 2012, 16.

[25] Bayoumi, Lee, and Jayanthi 2006, 272.

[26] Hummels, Ishii, and Yi 2001, 84; Breda, Cappariello, and Zizza 2008, 6.

[27] Breda, Cappariello, and Zizza 2008, Tab. 1-2.

many of the same major trading partners. In terms of trade, the two countries are very closely tied to the EU. Between 1997 and 2012, 63 percent of German imports and 71 percent of Belgian imports came from the EU, while 64 percent of German exports and 76 percent of Belgian exports came from the EU.[28]

Hypotheses and Tests

I test two broader hypotheses that will help determine whether the VAREER is a suitable replacement for the conventional REER for countries with high vertical specialization. The first hypothesis is that the VAREER performs better than conventional REERs as an explanatory variable of export demand for countries with high vertical specialization in trade. I study Belgium and Germany in order to test this hypothesis. Breda, Cappariello, and Zizza (2008) provide evidence that vertical specialization constitutes a high level of Belgian exports, and a low level of German exports.[29] This first conceptual hypothesis can be tested more directly using regression analysis by being broken down into the following four sub-hypotheses:

> *Hypothesis 1a: The Belgian VAREER will yield coefficients with a statistically-significant joint distribution as a regressor for Belgian exports.*
>
> *Hypothesis 1b: The German VAREER will fail to yield coefficients with a statistically-significant joint distribution as a regressor for German exports.*

[28] Author's calculation based on data gathered from the IMF's Direction of Trade Statistics.

[29] Breda, Cappariello, and Zizza 2008, Tab. 1-2.

Hypothesis 1c: The conventional Belgian REER will fail to yield coefficients with a statistically-significant joint distribution as a regressor of Belgian exports.

Hypothesis 1d: The conventional German REER will yield coefficients with a statistically-significant coefficient joint distribution as a regressor of German exports.

To confirm the theory of Bems and Johnson (2012) that high levels of vertical specialization in some countries make the VAREER more suitable than the REER, I should find that the VAREER is a more significant regressor than the REER for Belgium, the high vertical specialization country, and a less significant regressor than the REER for Germany, the low vertical specialization country. Hypothesis 1a and 1c test for whether the VAREER is superior to the REER in Belgium and 1b and 1d test for whether the REER is superior to the VAREER in Germany. Hypothesis 1c specifically tests the assertion by Bems and Johnson that the REER will be unsuitable for countries with high vertical specialization.

While the formulation of these sub-hypotheses may appear to create prohibitively strong statistical significance requirements to reach a definite conclusion, this is due to the nature of the testing and data. As of yet, there remains no systematic econometric means of comparing the relative significance of the joint distributions of different variables across different regressions of this type other than through direct comparison of the relative size of each of the F-statistics in question or through assessment of the statistical significance (or lack of statistical significance) of each individual joint distribution. Comparison through a simple panel regression pres-
ents a problem because the corresponding joint significance tests do not give information about which measure may be superior to another, only about whether there is a statistically-significant difference between the two. The inclusion of a euro interaction term attached to each REER only further com-

plicates comparison via panel regression. Thus, I have constructed the sub-hypotheses to be accommodating to the latter method, since clear differences in statistical significance can justify strong conclusions about the relative explanatory power of different coefficients. However, should the individual joint significance results not meet the stringent conditions specified above, I will also engage in direct comparison of the magnitudes of the F-statistics of each of the REER measures to see if they properly correspond to the results that the theory would predict.

Since the variables included in trade estimation equations traditionally tend to be co-integrated, Hypotheses 1a-d will be tested using an error correction model (ECM) based on the work of Engle and Granger (1987) to estimate the long-run and short-run effects of the conventional REER and VAREER on exports. The use of an error correction model here draws on the procedure of a wide range of papers dealing with trade estimation through ECMs, including Chowdhury (1993). Employing this method also reflects the frequently observed co-movements of trade time series, and is superior to a simple AR Distributed Lag (ARDL) regression since the ECM examines both long-run and short-run effects of the regressors on the dependent variable. In this case, the Engle-Granger test for co-integration is also preferable to the Johansen procedure because of the primary interest of this paper in the trade equation and the one-directional relationship between REERs and trade.

Since Bems and Johnson (2012) draw specific attention to the suitability of REERs for assessing export competitiveness, I analyze exports rather than imports.[30] For initial reference, I define the export demand function for each country to be as follows, drawing from Khan (1974):[31]

$$\log X_{it} = a_0 + a_1 \log(REER_{it}) + a_2 \log(W_t) + v_t \qquad (1)$$

[30]Bems and Johnson 2012, 2.
[31]Khan 1974, 682.

Here, for country i and time t, X represents export demand, W represents OECD real GDP, REER represents the real effective exchange rate, and v the error term. As a method of testing the hypotheses above, I replace Khan's relative export prices with the REER. This should not be problematic for either the conventional or value-added REER, as both consumer prices and GDP deflator take export prices into account. From this simple export demand equation, I then conduct the two-step Engle-Granger test for co-integration and construct an ECM to estimate the long-run and short-run effects of the REER on exports.

As Engle and Granger (1987) explain, multiple non-stationary series that are first-order integrated may become integrated of order zero if a stationary equilibrium relative to each other is formed when a linear combination of them is taken.[32] Such a relationship can then be reliably estimated using an ECM.[33]

Co-integration is dependent on the co-movements of the variables in question. The Engle-Granger method involves first estimating a long-run equilibrium equation of the variables of interest and then testing the residuals for stationarity using the Augmented Dickey-Fuller test.[34] If the residuals are found to be stationary, then we can be confident that the variables are co-integrated, and can estimate an ECM to analyze the relative significance of the REERs being tested.[35]

My first step is to confirm the order of integration of the variables to be tested. To do so I run an Augmented Dickey-Fuller test on the levels and first differences of the dependent variable and the regressors to confirm that the levels are integrated of order one and the first differences are stationary. The test is based on the following model:[36]

[32]Engle and Granger 1987, 253.
[33]Engle and Granger 1987, 254.
[34]Engle and Granger 1987, 264-267.
[35]Engle and Granger 1987, 275.
[36]Stock and Watson 2011, 553.

$$\Delta Y_t = b_0 + b_1 Y_{t-1} + \sum_{p=1}^{q} c_p \Delta Y_{t-p} \tag{2}$$

Here, Δ denotes the first difference operator. Y_{it} represents any time series variable. q is the optimal number of lags of the first differences, which I determine using Schwartz-Bayesian information criterion.[37] The null hypothesis of the ADF is $H_0 : b_1 = 0$, which is tested against the alternative that $H_a : b_1 < 0$.[38] If the Dickey-Fuller test statistic testing b_1 exceeds the critical value, then we can reject the null hypothesis that the series is non-stationary.[39]

Next, I use the Khan (1974) export demand function to construct four long-run equilibrium relationships, one regressed over the conventional REER and one regressed over the VA-REER for both Belgium and Germany. Each relationship also includes an interaction with a dummy variable for the Euro-zone period denoted by δ_{euro}, which takes on $\delta = 0$ before 1999Q1 and $\delta = 1$ after, since a large break in the exchange rate data occurs when Belgium and Germany adopt the euro. For country i and time t, the long-run relationships are written as follows:[40]

$$\begin{aligned}\log X_{it} = a_0 + a_1 \log((VA)REER_{it}) + \\ a_2 \log((VA)REER_{it})\delta_{euro} + a_3 \log(W_t) + v_t\end{aligned} \tag{3}$$

If the variables in the long-run relationship are co-integrated, the residuals should be stationary.[41] To test this, the second part of the Engle-Granger test regresses each residual over its lagged value:[42]

$$\Delta \hat{v}_t = b_0 + b_1 \hat{v}_{t-1} + u_t \tag{4}$$

[37] Stock and Watson 2011, 553.
[38] Stock and Watson 2011, 553.
[39] Stock and Watson 2011, 552.
[40] Chowdhury 1993, 701.
[41] Engle and Granger 1987, 275.
[42] Schaffer 2010.

Here, $\hat{v}_{t-1}$ represents the lagged residual. As with the Dickey-Fuller test for stationarity, the null hypothesis of non-stationarity, $H_0 : b_1 = 0$, is rejected when the test statistic of b_1 exceeds the critical value.[43] Rejection of the null hypothesis implies co-integration of the variables in the long-run relationship.[44]

The Engle-Granger ECM allows for an examination of the long-run and short-run effects of the REER on exports by regressing the dependent variable over the lagged residual (representing the long-run relationship), and the lagged first differences of OECD real GDP and the appropriate REER.[45] Thus the ECM takes the form below:[46]

$$\begin{aligned}\Delta \log X_{it} = \\ a_0 + a_1 \Delta \log(X_{i,t-1}) + a_2 \Delta \log(W_{t-1}) \delta_{euro} \\ + a_3 \Delta \log((VA) REER_{i,t-1}) \\ + a_4 \Delta \log((VA) REER_{i,t-1}) \delta_{euro} + \epsilon_t \quad (5)\end{aligned}$$

I test for Granger-causality on the coefficients of the value-added and conventional REERs in the short-run, and on the coefficient on the lagged residual of each regression, representing the long-run relationship. To be in line with the theory of Bems and Johnson (2012), I expect the Belgian VAREER and the German conventional REER to be "Granger causal," and the Belgian conventional REER and German VAREER to be "Granger non-causal."[47]

One shortcoming of the Engle-Granger ECM is that it fails to set optimal lags for the short-run lagged differences of the

[43] Stock and Watson 2011, 552.
[44] Engle and Granger 1987, 265.
[45] Engle and Granger 1987, 252.
[46] Chowdhury 1993, 703; Engle and Granger 1987, 262.
[47] Stock and Watson 2011, 538.

regressors. Thus, to get a more accurate picture of the short-run I construct four autoregressive distributed lag (ARDL) models using the stationary first differences of the log variables. Using the setup outlined by Stock and Watson (2011), I construct the ARDL models as follows:[48]

$$\begin{aligned}\Delta \log X_{it} =& a_0 + \sum_{\theta=1}^{n} a_\theta \Delta \log(X_{i,t-\theta}) + \sum_{\phi=1}^{n} a_{n+\phi} \Delta \log(W_{t-\phi}) \\ &+ \sum_{\omega=1}^{n} a_{2n+\omega} \Delta \log((VA)REER_{i,t-\omega}) \\ &+ \sum_{\alpha=1}^{n} a_{3n+\alpha} \Delta \log((VA)REER_{i,t-\alpha}) \delta_{euro} + v_t \end{aligned} \tag{6}$$

Here n denotes the optimal number of lags for each variable, with θ, ϕ, ω, and α representing the number of periods lagged in each instance. Using F-statistics on the coefficients of the value-added and conventional REERs, I test for Granger-causality on the predictive value of the total lags of the REERs and interaction terms.[49]

The key addition of the ARDL, the optimal number of lags, n, is selected using the Schwartz-Bayesian information criterion (SBIC). I regress the dependent variable several times over an increasing number of lags for each independent variable. The optimal number of lags is chosen from the regression that minimizes the following:[50]

$$SBIC(\theta) = \ln[\frac{SSR(\theta)}{T}] + \theta \frac{\ln(T)}{T} \tag{7}$$

θ is the number of lags in the regression, while T is the total number of observations in the sample. T remains fixed over the various regressions in order for the lag selection to be

[48] Stock and Watson 2011, 535.
[49] Stock and Watson 2011, 538.
[50] Stock and Watson 2011, 545.

accurate. I have chosen the SBIC over the Aikake information criterion (AIC) because the SBIC is more accurate in large samples, as the AIC tends to overestimate n on average.[51]

In order to thoroughly test the second conceptual hypothesis, which states that REERs constructed using value-added prices will explain fluctuations in exports better for countries with high vertical specialization they do for countries with low vertical specialization in trade, I plan to directly compare the joint significance of the VAREERs for Germany and Belgium using an autoregressive panel regression of exports over lags of world income and lags of the VAREER. The two sub-hypotheses associated with this panel regression test the joint significance of the VAREERs as predicted from the theory in Bems and Johnson (2012). They are:

> *Hypothesis 2a: The VAREER will yield coefficients with a statistically significant joint distribution when regressed over Belgian exports.*

> *Hypothesis 2b: The VAREER will fail to yield coefficients with a statistically significant joint distribution when regressed over German exports.*

This method offers a more systematic approach to control for any confounding factors or differences between the two countries not uncovered in the regressions addressing the first conceptual hypothesis. For country i and time t, the panel regression is constructed using the following setup:[52]

[51] Stock and Watson 2011, 544.
[52] Stock and Watson 2011, 356.

$$\begin{aligned}\Delta \log X_{it} =& \gamma_0 + \sum_{\theta=1}^{n} \gamma_\theta \Delta \log(X_{i,t-\theta}) + \sum_{\phi=1}^{n} \gamma_{n+\phi} \Delta \log(W_{t-\phi}) \\ &+ \sum_{\omega=1}^{n} \gamma_{2n+\omega} \Delta \log(VAREER_{i,t-\omega}) \\ &+ \sum_{\alpha=1}^{n} a_{3n+\alpha} \Delta \log(VAREER_{i,t-\alpha}) \delta_{BEL} \\ &+ \sum_{b=1}^{n} \gamma_{4n+b} \Delta \log(VAREER_{i,t-b}) \delta_{euro} \\ &+ \sum_{c=1}^{n} \gamma_{5n+c} \Delta \log(VAREER_{i,t-c}) \delta_{euro} \delta_{BEL} + z_i + e_t \end{aligned} \tag{8}$$

As above, Δ denotes the first difference operator and n constitutes the optimal number of lags determined by SBIC. First differences are used to satisfy the same stationarity arguments that are relevant with the ARDL model. For the panel regression, I use the same lags as the individual ARDL regressions for ease of comparison. The crucial interaction term for the panel is δ_{BEL}, which is equal to 1 when the data regressed belongs to Belgium, and is 0 otherwise. δ_{euro}, representing use of the euro as the national currency, is once again included as an interaction term with the VAREER of each country. Though Germany and Belgium share many important similarities, I also add a fixed effects term, z_i, as a precaution against further time-invariant differences between the two countries.

IV. Data

German and Belgian quarterly export data were drawn from the IMF's Direction of Trade Statistics (DOTS). As a proxy for quarterly world income, I use the aggregate real GDP of the select OECD members for which quarterly real GDP data are available over the entire period: Belgium, France, Finland, Germany, Norway, South Korea, Spain, Switzerland, the

United Kingdom, and the United States.[53] The real GDP data for these countries were pulled from Global Insight's Key Indicators.

For this paper, I manually constructed value-added and conventional REERs for both Belgium and Germany to ensure direct comparability. The REERs for Belgium and Germany are calculated using bilateral trade weights, nominal exchange rates, and relative price data for Belgium, Germany and their respective top trade partners. I define a "top trade partner" as any nation that accounted for at least 1 percent of the country's total imports or exports between 1980 and 2012.

For each trade partner and home country, quarterly nominal exchange rate data were pulled from the IMF's International Financial Statistics (IFS) database. Beginning in 1999Q1 the exchange rates of Austria, Belgium, France, Germany, Italy, the Netherlands, and Spain switched to the euro exchange rate.

Consumer price data came from the IMF's International Financial Statistics. However, due to fragmentation in the IFS German consumer price data, German consumer prices were collected from the OECD's Main Economic Indicators database. GDP deflator data for the countries above was drawn from the IMF's International Financial Statistics database using Global Insight.

I constructed the Bayoumi bilateral trade weights using commodities and manufactures trade data from the UN COMTRADE database. The trade weights were calculated annually for each bilateral trade relationship using the following formula:[54]

$$W_{ij} = a_M W(M) + a_C W(C) \tag{9}$$

Here, i denotes the home country (either Belgium or Germany) and j denotes the trade partner. a_M and a_C denote the overall shares of manufactures and commodities in global

[53] Global Insight. 1980-2012. "GDP, Real, US Dollars." Key Indicators.
[54] Bayoumi, Lee, and Jayanthi 2006, 279-280.

trade respectively, and W(M) and W(C) represent the partner-specific weights for manufactures and commodities trade with the home country.

The value-added and conventional REERs for Belgium and Germany were then constructed using the following method:[55]

$$(VA)REER_{ij} = \prod_{j \neq i} \left(\frac{P_i E_i}{P_j E_j}\right)^{w_{ij}} \tag{10}$$

The REER index (or VAREER index) is the geometric sum of the individual bilateral exchange rates weighted by bilateral trade exposure as defined in equation (9). For each home country i and trade partner j, P_i and P_j denote price level (using GDP deflator for VAREERs and CPI for REERs). E_i and E_j denote the bilateral nominal exchange rates with the United States, and W_{ij} denotes the bilateral trade weight for pair (i, j).

The VAREER analyzed in this paper differs from that of Bems and Johnson in two ways. First, as previously mentioned, I use a quarterly VAREER index rather than the annual index constructed by Bems and Johnson. This is done to make the VAREER more conducive to econometric analysis; while Bems and Johnson would have been limited to 39 observations in a regression using annual VAREERs, this paper is able to analyze 130 observations each for Belgium and Germany. Second, I elect not to use the Bems and Johnson (2012) value-added trade weights in constructing the VAREER. This is done to simplify the construction of the VAREER and is justified by the conclusion of Bems and Johnson that value-added trade weights do not account for the difference between the VAREER and REER.[56]

V. Results

Because export, GDP, and exchange rate data are traditionally non-stationary series, I test the stationarity of the levels and

[55] Bayoumi, Lee, and Jayanthi 2006, 286.

[56] Bems and Johnson 2012, 24.

first differences of each variable. Table 1 reports the results of the ADF tests.

Table 1. Augmented Dickey Fuller Test Results

TIMESERIES	LEVEL	FIRST DIFFERENCES
Log Bel Exports	-0.705	-4.509***
Log Ger Exports	-0.726	-5.081***
Log OECD GDP	-1.692	-3.857***
Log Bel REER	-2.579	-5.739***
Log Bel REER*Euro	-1.835	-8.051***
Log Bel VAREER	-2.337	-6.128***
Log Bel VAREER*Euro	-2.893**	-8.048***
Log Ger REER	-1.299	-5.807***
Log Ger REER*Euro	0.901	-7.998***
Log Ger VAREER	-1.393	-6.227***
Log Ger VAREER*Euro	-1.717	-8.034***

ADF test conducted on levels and first differences with intercept.
Test statistics for a null hypothesis of non-stationarity are reported.
***p<0.01, **p<0.05, *p<0.1

All of the variables tested except for the log of the Belgian VAREER-euro interaction term are identified as I(1) time series. This makes it possible to use the Engle-Granger two-step procedure to check for co-integration and construct an error-correction model that includes estimates for the long- and short-run relationships.

Table 2 shows the results of the Engle-Granger test for co-integration.[57] The null hypothesis of residual non-stationarity is rejected when the test statistic exceeds the critical value. If the residuals are found to be stationary, then exports, OECD real GDP, and the relevant REER are co-integrated.

[57]Schaffer 2010; Critical values reported by "egranger" come from MacKinnon (1990, 2010).

Table 2. Engle-Granger Test for Co-Integration

$Z(t)$	Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
Bel REER residual	-4.899***	-4.784	-4.182	-3.875
Bel VAREER residual	-4.899***	-4.784	-4.182	-3.875
Ger REER residual	-5.730***	-4.784	-4.182	-3.875
Ger VAREER residual	-3.781	-4.784	-4.182	-3.875

Regression type from which the residual is derived is specified.
Critical values come from MacKinnon (1990, 2010).
***p<0.01, **p<0.05, *p<0.1

These results reject the null hypothesis of no co-integration for the variables in both Belgian long-term export regressions and for the variables in the German VAREER regression. I fail to reject the null hypothesis of no co-integration for the German regression over the conventional REER; however, the test statistic falls less than 0.01 short of surpassing the 10 percent critical value. The presence of significant co-integration between the dependent and independent variables certainly justifies the inclusion of an error-correction model.

The first difference of each country's exports is regressed over the lagged first differences of exports, OECD real GDP, the VAREER or REER, and the error-correction term (i.e., lagged residual). Regressing the ECM yields the results listed in Table 3.

Table 3. Error-Correction Models

VARIABLES	(1) Belgian Exports	(2) Belgian Exports	(3) German Exports	(4) German Exports
Residual (-1)	-0.261***	-0.363***	-0.255***	-0.230***
	(0.0591)	(0.0636)	(0.0573)	(0.0572)
Log OECD GDP	-0.610	-1.108	-0.868	-0.705
	(1.078)	(1.024)	(0.894)	(0.903)
Log REER	3.733		1.454	
	(8.747)		(1.404)	
Log REER*Euro	-277.4		59.27	
	(330.3)		(49.48)	
Log VAREER		-5.285		0.0898
		(7.527)		(0.890)
Log VAREER*Euro		78.98		-0.0814
		(130.2)		(18.96)
Constant	0.0179*	0.0202**	0.0216**	0.0188**
	(0.0102)	(0.00958)	(0.00841)	(0.00860)
Observations	130	130	130	130
R-squared	0.137	0.229	0.141	0.118

Engle-Granger ECM of first differences.
Robust standard errors in parentheses.
***p<0.01, **p<0.05, *p<0.1

At first glance, a few interesting characteristics of the regressions stand out. The first is that all of the error-correction terms achieve statistical significance, while none of the short-run variables do so. The second is that the relative magnitude of the R-squared values exactly matches what would be predicted by the theory. The R-squared value of the Belgian export regression on the VAREER, 0.229, exceeds the R-squared value of the regression on the conventional REER, 0.137. Conversely, and as predicted, the R-squared value of the German export regression on the VAREER, 0.118, is exceeded by the R-squared value of the regression on the conventional REER, 0.141. This seems to favor the acceptance of the first conceptual hypothesis in the long-run. In terms of the second conceptual hypothesis, we also see that the R-squared value of the Belgian regression on the VAREER considerably exceeds the R-squared value of the German regression on the VAREER. Though these are separate regressions with different country data, these initial findings seem to support the second concep-

tual hypothesis that the VAREER will explain export demand variation better for Belgium, a country with high vertical specialization, than for Germany, a country with comparatively low vertical specialization.

To further analyze the ECM results, I conduct joint significance tests on the short-run value-added and conventional REER measures for each country, as well as on the error-correction terms representing the significance of the long-run relationships, and compare the results to test the first hypothesis. The results are found below in Table 4.

Table 4. ECM Joint Significance Comparison

VARIABLES	Belgium	Germany
Log REER-Log REER*Euro	0.37	0.72
Log VAREER-Bel VAREER*Euro	0.29	0.01
REER Residual (-1)	19.47***	19.72***
VAREER Residual (-1)	32.55***	16.11***

F(q, 125) statistics are reported as a test of joint significance.
q=number of variables tested
***p<0.01, **p<0.05, *p<0.1

The joint significance results from the ECM further corroborate the accuracy of the theoretical predictions with regards to the long-run relationship. While the long-run formulations of Hypotheses 1a-1d are all rejected under these conditions (whereas it was predicted that Hypothesis 1b and Hypothesis 1c would fail to be rejected) this failure can primarily be attributed to the stringent conditions that had to be imposed by necessity (since F-statistics have no standard error and as such no formal test for direct comparison of F-statistics between regressions of this type exists). However, an examination of the relative size of the F-statistics of the error-correction term for each regression yields observations consistent with the theoretical predictions of both conceptual hypotheses. The F-statistic of the Belgian VAREER exceeds the F-statistics for both the German VAREER and the Belgian conventional REER considerably. Furthermore, the F-statistics of the German conventional REER exceeds the F-statistic of the German VAREER.

These results appear to strongly suggest that, at least in

the long-run, the VAREER explains changes in export demand better than the conventional REER for countries with high vertical specialization. However, the short-run variables in the regressions are clearly not significant, and the p-values that correspond to several of their joint distributions approach 1. Part of the lack of significance may be due to the fact that the Engle-Granger ECM does not allow for optimal lag selection, and thus is less accurate an estimator of short-run effects than other models. As a solution, I estimate an ARDL model with optimal lags selected using the Schwartz-Bayesian information criterion. I also estimate a short-run fixed effects panel regression model to test H2a and H2b, since it is difficult to make confident statements about the second conceptual hypothesis through the ECM, as one must also account for any time-invariant cross-country differences that may be having an additional effect on the relative explanatory power of the German and Belgian regressions.

Given the non-stationarity of the levels of the log variables of interest, I estimate the ARDL using the stationary first differences of Belgian and German exports, OECD real GDP for select countries, and either the value-added or conventional REER for each respective country, where appropriate. To determine the optimal number of lags for each variable in the ARDL model, I apply the Schwartz-Bayesian information criterion for each of the first-differenced log variables, regressing exports on lags of itself and each of the independent variables. I choose to conduct the process for Belgium alone for ease of direct comparison of the models. This is justified since the German time series have similar characteristics. Table 5 reports the results of the Schwartz Bayesian Criterion lag selection.

Table 5. Schwartz-Bayesian Information Criterion

	Lags					
VARIABLES	1	2	3	4	5	6
Belgian Exports	-248.1	244.6	-257.6	-292.3	-293.0*	-285.5
OECD GDP	-240.8	-251.0	-258.7*	-256.6	-250.7	-245.7
Belgian REER	-240.8*	-236.1	-229.1	-224.0	-216.5	-210.8
Belgian VAREER	-240.9*	-236.2	-229.7	-224.5	-217.0	-211.7

SBIC performed on first differences..
*denotes lowest SBIC statistic

In line with the results in Table 5, I regress an ARDL(5,3,1,1) for each of the four REER measures and compare the F-statistics as prescribed by the hypothesis tests in Section 3.2. Table 6 shows the results of the ARDL regressions.

Table 6. ARDL Regressions

VARIABLES	(1) Belgian Exports	(2) Belgian Exports	(3) German Exports	(4) German Exports
Exports (-1)	-0.0560	-0.0530	-0.0367	-0.0378
	(0.0878)	(0.0880)	(0.0977)	(0.0985)
Exports (-2)	-0.153**	-0.162**	-0.0546	-0.0529
	(0.0737)	(0.0740)	(0.0919)	(0.0936)
Exports (-3)	-0.311***	-0.303***	-0.153	-0.154
	(0.0820)	(0.0816)	(0.108)	(0.108)
Exports (-4)	0.389***	0.389***	0.303***	0.301***
	(0.0766)	(0.0770)	(0.0733)	(0.0742)
Exports (-5)	-0.170*	-0.173*	-0.181*	-0.180*
	(0.0890)	(0.0897)	(0.107)	(0.107)
OECD GDP (-1)	2.719**	2.685**	2.321**	2.304**
	(1.052)	(1.059)	(1.057)	(1.047)
OECD GDP (-2)	2.215***	2.267***	2.201**	2.198**
	(0.819)	(0.818)	(0.969)	(0.969)
OECD GDP (-3)	-1.748	-1.754	-2.448**	-2.429**
	(1.189)	(1.193)	(1.161)	(1.162)
REER (-1)	-30.23*		1.478	
	(17.74)		(3.109)	
REER*Euro (-1)	38.42**		-2.005	
	(17.82)		(3.121)	
VAREER (-1)		-25.08		1.093
		(16.28)		(3.203)
VAREER*Euro (-1)		32.19*		-1.617
		(16.25)		(3.236)
Constant	0.00333	0.00322	0.00590	0.00582
	(0.0107)	(0.0107)	(0.0114)	(0.0114)
Observations	126	126	126	126
R-squared	0.530	0.530	0.369	0.368

OLS regression of lags of first differences.
Robust standard errors in parentheses.
***p<0.01, **p<0.05, *p<0.1

A few characteristics of the regressions are noticeable at first glance from the output. Namely, we see that the R-squared values of the Belgian regressions are essentially the same, at .530, truncated at the thousandths place. Likewise, the R-squared values for the German regressions differ only by .001. However, the R-squared values are substantially lower than those of the Belgian regressions. This indicates that there may be a time-invariant difference between the two countries for which the ARDL model does not control. Thus, the addition of a panel regression analysis will be beneficial.

Using the regression output in Table 6, I compare the joint significance of the variables to test Hypotheses 1a-d. The results of the F-tests are found in Table 7.

Table 7. ARDL Joint Significance Comparison

VARIABLES	Belgium	Germany
REER (-1)	2.90*	0.23
REER (-1)*Euro	4.65**	0.41
REER (-1), REER (-1)*Euro	24.10***	6.08***
VAREER (-1)	2.37	0.12
VAREER (-1)*Euro	3.92*	0.25
VAREER (-1), VAREER (-1)*Euro	24.79***	5.51***

F(q, 116) statistics are reported as a test of joint significance.
q=number of variables tested
***p<0.01, **p<0.05, *p<0.1

It is immediately clear that the joint significance tests reject all four of the short-run sub-hypotheses, as opposed to rejecting only H1a and H1d, as the theory suggested. Thus, the ARDL analysis is not in a position to completely endorse the suitability of the VAREER as a replacement for the conventional REER for countries with high vertical specialization, though much of the reason why, again, is due to the lack of systematic means of comparing different F-statistics across regressions. However, as was also true with the ECM, the magnitude of the F-statistics are, in every case, consistent with the expectations of the theory. In the Belgian regression, the joint significance of the VAREER, with an F-statistic of 24.79, is greater than the joint significance of the REER, at 24.10. Likewise, in the German regression, the joint significance of the VAREER is less than the REER, at 5.51 and 6.08 significance respectively. One should note that these differences in magnitude are much smaller than the corresponding long-run differences, which were pronounced. Nevertheless, the relative sizes of the F-statistics do support the theory.

The size of the spread between the Belgian and German R-squared values and F-statistics, in addition to the need to test Hypotheses 2a-b, further necessitates the estimation of the fixed effects panel regression. Using the same lags as the ARDL model, I regress exports over OECD real GDP and

the VAREER rates in a panel regression. An interaction term is included to distinguish between the Belgian and German VAREERs. Running the panel regression with fixed effects results in the output summarized in Table 8.

Table 8. Fixed Effects Panel Regression

	(1)
VARIABLES	Exports
Exports (-1)	-0.0312
	(0.0637)
Exports (-2)	-0.114**
	(0.0571)
Exports (-3)	-0.237***
	(0.0555)
Exports (-4)	0.353***
	(0.0549)
Exports (-5)	-0.180***
	(0.0600)
OECD GDP (-1)	2.315***
	(0.689)
OECD GDP (-2)	2.296***
	(0.667)
OECD GDP (-3)	-2.056***
	(0.705)
Ger VAREER (-1)	-0.486
	(0.694)
Ger VAREER (-1)*Euro	145.0
	(101.3)
Bel VAREER (-1)	-24.20
	(18.45)
Bel VAREER (-1)*Euro	26.73
	(19.57)
Constant	0.00456
	(0.00687)
Observations	252
R-squared	0.459

Robust standard errors in parentheses.
***p<0.01, **p<0.05, *p<0.1

To address the validity of the null hypotheses, I calculate the joint significance of the German and Belgian VAREERs.

The F-statistics for their joint distributions are summarized in Table 9.

Table 9. Panel Regression Joint Significance

VARIABLES	Belgium	Germany
VAREER (-1)	1.72	0.49
VAREER (-1)*Euro	1.87	2.05
VAREER (-1), VAREER (-1)*Euro	0.93	1.28

F(q, 238) statistics are reported as a test of joint significance.
q=number of variables tested
***p<0.01, **p<0.05, *p<0.1

The panel regression yields F-statistics that contradict the joint significance results of the ARDL model. Not only are none of the VAREERs statistically significant, where it would be expected that at least the Belgian VAREER be, but the test statistic of the German VAREER, 1.28, exceeds that of the Belgian VAREER, at 0.93. The contradiction between the results of the panel regression and the theory-supporting results of the ARDL is certainly intriguing, especially since one might be inclined to believe that a fixed effects regression would improve the accuracy of the results. At the very least, since this panel regression is a short-run model, the results support the initial finding of statistical insignificance of the short-run relationship in the ECM. Thus, the panel results should cast doubt on the relative and absolute sizes of the joint significance test statistics in the ARDL model. But, importantly, the results do not directly contradict the findings of a significant long-run relationship in the ECM and actually corroborate the weakness of the short-run variables in the ECM.

VI. Conclusion

The rise in vertical specialization as a phenomenon in world trade indicates that REERs constructed to reflect value-added trade and value-added prices might be more suitable for explaining changes in a country's export volume if its export structure is characterized by high levels of vertical specializa-

tion. Bems and Johnson (2012) construct such an exchange rate, and observe substantial differences between VAREERs and conventional REERs, determined primarily by the replacement of consumer prices with GDP deflator as a proxy for value-added prices, and not by the modification of bilateral trade weights to reflect value added trade.[58]

This paper tests the suitability of the replacement of consumer prices with GDP deflator by constructing conventional and value-added REERs for two countries with different levels of vertical specialization: Germany and Belgium. I estimate an error-correction model, as prescribed by Engle and Granger (1987), to compare the relative explanatory strength of the value-added and conventional real effective rates for Germany and Belgium. The results of the ECM indicate that, in the long-run, the VAREER surpasses the ability of the conventional REER to explain export demand for Belgium, while the opposite is true for Germany, as expected.

The ECM also shows, however, that the value-added and conventional REERs have negligible short-run explanatory power. It was suggested that part of this may be due to the inability of the ECM to take into account optimal lags for each variable. To investigate this possibility, I regress an ARDL model using first-differences to estimate Belgian and German exports with lagged exports, lags of OECD real GDP, and lags of the two different REERs. The results of the ARDL regressions indicate that the value-added and conventional REERs are significant for both countries in the short-run, and the relative size of the F-statistics of the different rates correspond with the predictions of the theory; specifically, the joint significance of the VAREER is slightly greater than that of the conventional REER for Belgium, while the VAREER significance is slightly lower than that of the conventional REER for Germany.

The ARDL results, though, also suggest that there may be differences between the two countries not properly controlled for in the initial regressions. As a further robustness check, I estimate a fixed effects panel regression of the VAREER re-

[58] Bems and Johnson 2012, 24.

gressed over exports with the country as the panel variable. The results of the panel regression cloud the picture provided by the ECM and ARDL model. The panel joint significance tests suggest that the VAREER might not be able to claim statistical significance as a predictor of exports for either country in the short-run. Furthermore, the relative magnitude of the F-statistics in the panel slightly contradicts the relationship predicted by the theory as well as the findings of the ARDL model, since the joint significance of the German VAREER exceeds that of the Belgian VAREER in the panel. The panel results, however, are consistent with the findings of the ECM, which also indicated negligible short-run effects in the export equations for both countries.

To address the failure to find conclusive results in the short-run, there are typically three reasons why the quantitative predictions of a theory are not visible in the data: (i) the effect is too small, (ii) the theory is wrong, or (iii) the methodology is imperfect/not very powerful. Since the long-run results in the ECM strongly corroborate the theory put forward by Bems and Johnson (2012), it seems unlikely they have made a theoretical error. However, there may be merit in explanations (i) and (iii).

With regards to the first explanation, one potential issue that may explain the failure of the panel and short-run ECM estimates to distinguish between the German and Belgian rates in a manner consistent with the theory is that the level of vertical specialization in each country likely varied at different rates during the period studied. This would certainly have a substantial effect on the findings of the short-run models. Furthermore, the scarcity of more recent data on vertical specialization makes it difficult to completely ascertain the relative size differences in the level of vertical specialization between Belgium and Germany over the entire period.

With regards to the third explanation for the failure to find conclusive short-run results, the lack of an error-correction model that uses optimal lags in the short-run may account for the statistical insignificance of the first differences of the

REERs. To fix this, future work with an ARDL error-correction model, which contains short-run variables consistent with the long-run relationship and chooses optimal lags for the short-run, may give a more accurate picture of the short-run effect of REERs on export demand.

In sum, the results of the Engle-Granger ECM are promising for the long-run validity of the claims of Bems and Johnson (2012). However, since the panel regression seems to call into question the short-run results of the ARDL model, the VAREER may have negligible utility as a policy tool for assessing competitiveness, since the short-run relationship holds greater weight in policy deliberations. Further research that looks into the long- and short-run suitability of the VAREER in other contexts would be valuable in helping to clarify these issues. Specifically, expansion of the work done in this paper using ARDL and panel error-correction models that account for optimal lags in the short-run could provide a more accurate picture of the value-added rate's suitability over both periods. Furthermore, as more data on vertical specialization become available, extending these techniques to a broader range of countries to get a more comprehensive view of the differences between the value-added and conventional REERs would represent another important contribution. Additional research that more directly tests for a connection between the suitability of the VAREER and rising vertical specialization by incorporating continuous measures of vertical specialization in place of using countries with different levels of vertical specialization as an indirect test of the relationship would also be of value.

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China and India in Africa: Implications of New Private Sector Actors on Bribe Paying Incidence

Sankalp Gowda

Abstract

This paper seeks to address one of the most common critiques of Asian firms doing business in Africa: that low levels of corporate governance and poor managerial practices have undermined anti-corruption efforts throughout the continent. The paper first details and analyzes the managerial practices of Indian and Chinese firms to distinguish what factors might make these firms more likely to pay bribes. Next, it uses data from the 2006-2014 World Bank Enterprise Surveys to empirically test the claim that the presence of Indian and Chinese firms has increased bribe-paying incidence in African countries.??I find the result that firms operating in countries with large Indian and Chinese involvement are significantly less likely to engage in bribe paying. This is promising evidence against the "race to the bottom" scenario that many Western firms and governments have complained of in response to the growing Asian presence in Africa.

I. Introduction

Over the course of the past decade, African nations have increasingly adopted a new view toward development that focuses on bolstering private sector growth through investment and trade. Driven by mounting criticism of the effectiveness of foreign aid dollars and cynicism toward the sustainability of development priorities set by Western nations, this move has coincided with the rise of India and China as global economic powers. Indian and Chinese firms have stepped in to fill the investment gap left in emerging economies by more cautious Western investors, and have heavily prioritized building South-South relationships over the past several years. The economic significance of this trend is remarkable. The global south was

responsible for 34% of all foreign direct investment to the developing world in 2010 and China's outward FDI to the south alone totals over $1 trillion (Puri, 2010; World Bank, 2011, 23). Additionally, South-South exports grew to $4 trillion in 2011 and increased from 13% of world exports in 2001 to 25% in 2011 (UNCTAD, 2013, 1). Although not all of these flows have been directed at African nations, they have undoubtedly played a significant role in contributing to the continent's 4% growth rate in 2013, helping Africa top the global average of 3% (African Economic Outlook, 2014).

Understandably, this growth rate and these trends do not extend to all African nations, and Indian and Chinese involvement has been limited to a number of key countries. Primary among these are the oil and mineral rich Nigeria, Sudan, and Zambia that are critical to meeting Indian and Chinese demand for resources, but others include Botswana, Ethiopia, Kenya, Madagascar, Mauritius, Mozambique, Senegal, South Africa, and Uganda where investment has extended into a much broader range of sectors (Broadman, 2008). Although many applaud Indian and Chinese firms for boosting competition, providing access to new global supply chains, and producing learning effects through technology and knowledge transfers, their reception in these countries has been mixed. Common critiques of foreign firms from Asia include the undercutting of local wages/exclusion of the local labor market, quality concerns over working conditions and outputs, and the central focus of this paper: low levels of corporate governance that could undermine anticorruption efforts. This last view is well represented by Western donors and businesses. During a 2012 speech in Senegal, former Secretary of State Hillary Clinton took an indirect jab at India and China when she stated, "The USA stands for democracy and human rights, even when it's easier or more profitable to look away in order to secure resources." (Deutsche Welle, 2012). Business leaders echo this sentiment and point to anticorruption legislation such as the United States's Foreign Corrupt Practices Act and the lack of similar legislation in India and China as an inherent dis-

advantage for Western firms (Wall Street Journal, 2014). The implication is clear: Indian and Chinese firms have been eclipsing Western investors through bribe paying and other corrupt practices. While India and China have been quick to refute such claims, they continue to cast a shadow over further investment efforts. Notably, much of this criticism has been directed toward Chinese firms while Indian firms have for the most part escaped relatively unscathed.

This paper seeks to deconstruct the impact of Indian and Chinese firms as new private sector actors on bribe paying incidence in the African region. Section 1 describes important differences in each nation's approach to investing in Africa. Section 2 discusses how these differences might affect the supply-side of corruption through management practices that tolerate or even embrace bribery. Section 3 summarizes literature regarding institutional drivers of corruption at the firm level that theoretically affect all firms operating in Africa. Section 4 outlines the data and methodology of my empirical analysis and Section 5 presents preliminary empirical findings from the World Bank Enterprise Surveys. I conclude with policy implications and suggestions for areas of further research.

I find that evidence from the Enterprise Surveys supports the surprising result that firms operating in countries with large Indian and Chinese involvement are significantly less likely to engage in bribe paying. However, this result might be driven more by institutional environment than by firm activity. This evidence alone is not enough to exculpate Indian and Chinese firms specifically from any wrongdoing, but it is promising evidence against the "race to the bottom" hypothesis that has been raised against foreign firms operating in Africa. Ultimately, more detailed data will be required to conclusively gauge the impact of Indian and Chinese firms on corruption in the African private sector.

II. The Indian vs. Chinese Approach to FDI in Africa

Of the two countries, China remains the dominant player in Africa, with approximately $119.7 billion dollars in FDI outflows to the continent between 2007 and 2012 compared to India's $27.3 billion (Fortin, 2013, Ernst Young, 2013). Although these numbers still lag far behind those of Western countries like the United States, United Kingdom, and France, China has managed to become Africa's largest trading partner, surpassing the United States in 2009. The majority of China's investments are in resource intensive industries, particularly oil and natural gas, as it depends heavily on Africa for its energy needs (approximately one-third of its crude oil comes from Africa). Its investments in these industries have been accompanied by large-scale infrastructure projects in roads, ports, and buildings, adding to its visibility on the continent (Alessi Hansion, 2012; Khare, 2013). The size of these contracts also means that the majority of these investments are made at the state-state level through state-owned enterprises (SOEs) or sponsored by state agencies such as China's ExIm Bank (The Economist, 2011). Starting in 2010, this trend has shifted toward a more diversified set of industries, with transportation, agriculture, and real estate investments eclipsing natural resource investment (Caulderwood, 2014). China's centralized approach to investment on the continent has been backed by high-level visits from President Xi Jinping in 2013 as well as by former President Hu Jintao throughout the 2000s. As a result of its growth and heavy involvement in Africa's capital-intensive industries, China has garnered more attention than any other investor in recent years.

India, on the other hand, operates by virtue of a very different model. Although its state-owned energy companies pursue India's interests in African resource markets in the same manner as China's, the majority of its investment in Africa is led by the private sector (Jacobs, 2013). This is reflected by the fact that although India's FDI outflows were only one-fifth of China's, India was responsible for 56% more new FDI

projects than China between 2007 and 2012 (Ernst Young, 2013). These projects represent a far more diversified portfolio of smaller investments than China's, and India is known for its presence in a broader range of sectors. These sectors include agriculture, IT, telecommunications, and healthcare/medicine. Interestingly, these are sectors that avoid direct competition with Chinese investments in Africa, a product of private financing and a more traditional program of risk assessment spurred by a lack of central state backing. Among the largest private sector actors in the Indian expansion into Africa are globally well-known and regarded firms such as the Tata Group, Godrej, and Bharti Airtel (Indo-African Business Magazine, 2011).

Indian and Chinese firms are further differentiated by their level of integration within local African economies (The Econ omist, 2013b). The Indian presence in East Africa has existed for more than a century, and the two regions are bound by a common colonial legacy. Furthermore, the strong, integrated Indian diaspora serves as a natural base to promote Indian interests in the region. This translates to a smaller language and culture barrier than that faced by Chinese firms. Additionally, a survey of Indian business leaders actively investing in Africa undertaken at the most recent WEF India Economic Summit in 2014 reported a hiring target for local employees of 90% and a new push to produce certain types of products in Africa instead of focusing on selling finished goods (Vanham, 2014). Chinese involvement has meanwhile been seen as the more foreign of the two and has at times evoked a xenophobic response from local populations. Some countries such as Malawi, Tanzania, Uganda, and Zambia have responded by restricting the sectors in which Chinese firms can operate. This suspicion is in part due to protectionism by African businesses, but it remains one of China's biggest hurdles in Africa (The Economist, 2013a). Even more serious incidents such as recurring riots over working conditions in Zambian mines also continue to color popular perception of China as the continent's new neocolonial power.

III. The Supply-Side of Corruption

It is clear that Indian and Chinese firms have vastly different approaches to doing business in Africa. Deconstructing these differences further could provide insight into how their management practices might affect the supply-side drivers of corruption. Given that the Western critique of Indian and Chinese firms is focused here, this is a subject worth exploring despite a paucity of existing literature.

In mainland China, it is well documented that bribe paying and other forms of corruption are common business practice, reflected by China's rank as 100th out of 175 countries on Transparency International's Corruption Perceptions index (Transparency International, 2014). The business culture is reliant upon relationships and "gifts," even in the private sector. Cai, et al. (2011) use a conceptual management theory of the Chinese firm to empirically show that entertainment and travel costs, a common category in most Chinese firms' financials, is often used as a proxy for dollars spent on bribe paying or other similar activities (Cai, et al., 2011) . Similarly, anecdotal evidence from Chinese firms in Africa supports the theory that these practices have been exported overseas. Chinese managers have been documented bribing union bosses with fake "study tours" to China to avoid censure over poor working conditions (The Economist, 2011). Additionally, there are numerous cases of Chinese SOEs like Nuctech Company - at one point managed by President Hu Jintao's son - becoming the subject of both African and European anti-corruption probes (Gordon, et al., 2009). While the majority of these cases are on a smaller scale, a recent incident with Sicomines, a Chinese state-owned mining company, gives a better sense of the how much money can change hands in one of these transactions. The company recently signed a $6.5 billion deal with the Democratic Republic of Congo that included a $350 million "signing bonus." According to accountability NGO Global Watch, $24 million of this bonus made its way to secret bank accounts in the British Virgin Islands rather than into the DRC's treasury (Kushner, 2013).

India, at 85th on Transparency International's Corruption Perceptions Index, is not necessarily a much better contender for clean management practices in its own private sector (Transparency International, 2014). Aside from a more limited relationship with the state and a more traditional business perspective on risk management that might preclude firms from entering a corrupt market where costs are higher, there is little to indicate that Indian firms are less likely to engage in bribe paying than Chinese firms. Interestingly, however, a 2013 Transparency International study of BRICS firms operating in emerging markets ranked Indian firms first in transparency while Chinese firms ranked last. The rankings cited key Indian laws requiring publication of certain financial information as the driving force behind the relative transparency of Indian firms. Additionally, with more publicly listed companies on the list, Indian firms performed better than other BRICS nations that had more state or private-owned firms. Publicly listed companies are more accountable to shareholders and typically have more disclosure requirements. Tata Communications, the Indian firm that topped the list, also incorporated several additional measures into its corporate governance structure that included bribe reporting and whistleblower protection (Gayathri, 2013).

However, Transparency International's rankings should not mask the fact that even Indian firms are far from perfect. Mining conglomerate Vedanta, which operates globally and has multiple investments in Africa, was found guilty of rampant corruption in India throughout the 2000s (Rankin, 2013). These practices may very well be replicated in Africa; as recently as 2011, Vedanta acquired a Liberian iron ore company that was being investigated by the Liberian anti-corruption committee. Such deals demonstrate the low emphasis that some Indian firms like Vedanta place on corruption in their risk assessment practices (Financial Times, 2011). Even Bharti Airtel, which ranked fourth on Transparency International's list of most transparent firms, is currently facing charges of corruption in India over suspicious dealings with former Tele-

com Minister Andimuthu Raja. Since 2013, the chairman of Bharti Airtel has refused to answer his summons to testify in the case and has escalated the issue of his appearance to the Indian Supreme Court (Rautray, 2014). Although this evidence is only anecdotal, it is a good indication that management practices even among large publically traded Indian firms may mirror those of China's state-owned enterprises.

From the supply-side perspective, both Indian and Chinese management practices appear to incorporate bribery and similar tactics in spite of numerous domestic anti-corruption laws. As two countries that rank relatively low on the Corruption Perception Index, this is not altogether surprising. However, in terms of how these practices transfer overseas, it is important to recognize that Indian and Chinese firms may not always perform worse than Western firms. Returning to the example of mining in the Democratic Republic of the Congo, the 2009 COMIDE deal is an example of how questionable circumstances led to the sale of the DRC's 25% stake in a copper mining venture to a European multinational headquartered in London (Kushner, 2013). DRC officials who signed the deal failed to disclose the sale to the public, in violation of a conditional development loan from the International Monetary Fund. Ultimately, the IMF declined to renew its loan as a direct result of the COMIDE incident and the DRC forfeited valuable development funds. Western firms might face greater regulation but this does not always translate to more reliable accountability.

IV. Firm-level Determinants of Corruption

In addition to the supply-side determinants of corruption, bribery is also a result of the institutional investment climate in the countries where firms operate. Fitting within the traditional definition of corruption as public officials' abuse of their office for private gain, the demand side of corruption allows us to identify firm-level characteristics for which firms are asked to pay bribes. In the empirical analysis that follows this section, these firm-level determinants will serve as a baseline to gauge if

foreign firms are more likely to pay bribes in Africa as a whole, and to provide a rough estimation of the potential supply-side impact of Chinese and Indian firms in the countries where they operate.

There are three primary hypotheses regarding firm-level determinants of corruption in the existing literature: the Control Rights hypothesis, the Bargaining Power hypothesis, and the Grease the Wheels hypothesis. Control Rights is based most heavily on the definition above, and focuses on public officials' opportunity to extract bribes. A firm's required dealings with the public sector for services such as water and electricity determine the firms' dependency on public officials and its exposure to corruption risk (Svensson, 2003). By this logic, a firm that is more frequently in contact with the public sector is at increased risk of needing to pay a bribe. Bargaining Power refers to a firm's position to refuse paying the bribe, quantifiable by its relative cost of exiting the market. If the cost of paying the bribe is greater than the firm's cost of exiting, firms can more credibly refuse to pay the public officials (Svensson, 2003). This also holds in the opposite sense that stronger performing firms, i.e. those that are more profitable or solvent, will face more solicitation for bribes from savvy corrupt public officials. Lastly, Grease the Wheels refers to a mixed supply/demand explanation for bribe paying where firms bribe in order to circumvent or speed up procedures in an otherwise burdensome administrative environment (Alaimo, et al., 2009). Firms that are in this situation (i.e. our foreign Chinese and Indian firms) will pay bribes to gain an advantage over competitors or simply to respond to inefficient institutions in the operating country.

Because these institutional determinants of corruption theoretically extend to all private sector actors operating within the same industry and country/region, they serve as a good baseline lens through which to view corruption. The empirical evidence on each of these hypotheses is mixed, and varies based on the level of analysis - country, regional, or global. Several key examples include Svensson (2003), Alaimo, et al. (2009),

and Chen, et al (2008). Svensson (2003) tests the Control Rights and Bargaining Power hypotheses using survey data from Ugandan firms, and finds that both are powerful predictors of not only which firms pay bribes, but also how much they must pay. Alaimo, et al. (2009) test all three hypotheses at the regional level for Latin American firms and find support for Control Rights and Grease the Wheels, but do not find evidence in support of Bargaining Power. Chen, et al. (2008) conduct a cross-country analysis that incorporates the first two hypotheses (and implicitly the third) as well as several macro-level determinants of corruption. The authors find that certain firm characteristics, such as dependence on infrastructure, likelihood of going to an alternative authority, and number of competitors, are significant determinants of corruption that function similarly to the three hypotheses. They also find that certain macro-level determinants such as British legal origin and average years of schooling are significantly correlated with lower levels of firm-level corruption, while population is a significant and positive determinant of corruption. The following empirical section will draw primarily on the techniques used by these authors as applied to the World Bank Enterprise Surveys Standardized dataset from 2006-2014.

V. Conceptual Framework, Data Description, and Empirical Specification

As discussed in the above studies (see Svensson, 2003 and Chen, et al., 2011), the factors that affect bribe payout by firms can be expressed as a function of several different factors:

$$Br_{ij} = f(X_j, c_i, b_i, g_i, z_i) \tag{1}$$

where Br_{ij} is the amount of bribes paid out by firm i in country j, X is a vector of country level attributes representing culture, legal systems, and institutional capacity; c is a vector of firm level characteristics representing Control Rights; b is a vector of firm level characteristics representing Bargaining

Power; g is a vector of firm level characteristics representing Grease the Wheels; and z is a vector of other firm characteristics (unrelated to the three hypotheses) that might also lead to bribe paying. The first set of vectors is macro-level while the second set is focused at the firm level.

For the sake of this analysis, I will set aside the level of bribe payouts and instead look at bribe paying incidence - whether firms report having paid any bribes to a public official - as the dependent variable. This can be expressed as:

$$BD_{ij} = f(X_j, c_i, b_i, g_i, z_i) \tag{2}$$

where BD is a dummy variable equal to one if the firm reports paying a bribe, and zero otherwise. The dependent variable comes from several questions in the Enterprise Surveys which ask the respondent whether a "gift or informal payment" was expected or requested with regard to customs, taxes, licenses, regulation, public services, etc.

In addition to the dependent dummy variable, the other firm level variables also come from the Enterprise Surveys. The World Bank Enterprise Surveys provide a cross-sectional survey of industrial and service enterprises, with the data used in this analysis focusing on the Africa region between the years of 2006 and 2014. Data collection efforts were led by the World Bank, which has been administering business environment surveys since the mid 1990s. The surveys focus on the manufacturing and services sectors and 100% state owned enterprises are not allowed to participate. Important for the purposes of this paper, the surveys also do not include data from firms operating in extractive industries like oil or minerals. The surveys are administered through face-to-face interviews with business owners and top managers (World Bank, 2014).

The firm level vectors use variables that I created from responses to the Enterprise Surveys. The Control Rights vector is represented by the Government Help dummy variable, which is equal to one if a firm requested any public services in the past two years. According to the theory above, requesting government help is expected to have a positive relationship with

bribe paying. The Bargaining Rights vector is represented by two dummy variables: Access to Credit and Credit Constrained. Access to Credit is used to gauge a firm's solvency, and is equal to one when firms have access to a line of credit or overdraft facility. Credit Constrained is used to gauge how difficult it would be for a firm to pick up and move to a less corrupt market, and is equal to one when firms have a) applied for a loan and been rejected, or b) not applied for a loan for reasons other than "does not need a loan." Both of these firm traits are expected to have a positive relationship with bribe paying. Grease the Wheels is measured through two dummy variables: Trust in Courts and Competition. Trust in Courts measures firms' belief in the effectiveness of government regulation and bureaucracy, and is equal to one when respondents said they believed the judicial system worked fairly and impartially. Competition measures the business environment in which firms are operating, and is equal to one if firms reported reducing prices due to competition against another firm. Trust in Courts is expected to have a negative relationship with bribe paying and Competition is expected to have a positive relationship as firms make decisions to gain an advantage over their competitors. I also created a Foreign dummy variable which equals one if the firm has any foreign ownership. This last variable will provide some insight to the impact of foreign firms on corruption in Africa but data limitations prevent us from separating Indian and Chinese firms from the rest.

Other firm level variables include Registered (=1 if the firm was officialy registered when it began operations), Government Owned (=1 if any government ownership), Medium (=1 if the firm has 20-99 employees), Large (=1 if the firm has greater than 100 employees), Young (=1 if the firm has operated for less than 20 years), Old (=1 if the firm has operated for more than 50 years), Sales (the log of last year's sales), and Trade (=1 if the firm imported or exported any goods).

Chen, et al. (2008) includes several macro-level variables, but I decided to focus on the two in particular that I felt were of the most importance to this paper. The first is a British

Legal Origin dummy variable, which I adapted from a list of countries with British legal origins found in Klerman, et al (2012). For the African continent, this includes Ghana, Tanzania, Malawi, Uganda, Gambia, Zambia, Nigeria, Kenya, Mauritius, Lesotho, South Africa, and Zimbabwe. The second is an IndoChina dummy variable, which I set equal to 1 for African countries that have developed strong investment and trade relationships with India and China (Broadman, 2008; Dahman-Saãdi, 2013; Leung and Zhou, 2014; Nayyar and Aggarwal, 2014, 2). Countries coded for the IndoChina dummy are South Africa, Nigeria, Zambia, Algeria, Sudan, DRC, Ethiopia, Mauritius, Tanzania, Madagascar, Guinea, Kenya, Mozambique, Senegal, and Uganda. If Indian and Chinese firms have in fact exported their bribery-heavy management practices to these countries, this variable should be positively related to corruption. To further disaggregate this potential result, I also created an interaction term called IndoChina x Foreign to see the effect of being a foreign firm operating in a country with a strong IndoChina presence. If the claims of Indian and Chinese corruption are to be believed, this term should bear a strong positive relationship to bribe paying.

Within this particular mix of variables, there is the potential that some micro and macro variables could fall under each of the vectors on the right-hand side of equations (1) and (2). To ensure a proper model and avoid incorrect inferences due to multicollinearity, I constructed a correlation matrix (See Appendix Table 1) for all independent variables in the dataset. I then dropped variables with particularly high correlations ($>.50$) that could create multicollinearity issues. For example, I did not include IndoChina and British Legal Origin in the same specification, although it would have been interesting to see the effect of controlling for legal origin on the IndoChina coefficient.

After taking these results into account, I developed the following basic economic specification that I adapted for four different models:

$$BD_{ij} = \beta_1 GH + \beta_2 AC + \beta_3 CC + \beta_4 R + \beta_5 F + \beta_6 TC + \beta_7 C + \beta_8 GO + \beta_9 M + \beta_{10} L + \beta_{11} Y + \beta_{12} O + \beta_{13} S + \beta_{14} T + \beta_{15} IC + \beta_{16} ICF + \beta_{17} BLO \quad (3)$$

The summary statistics for these variables are included in Table 1. For the sake of brevity, I will not go into detail regarding the estimation procedure, but the basics are as follows: I used a probit regression model to account for the binary choice faced by firms in determining the dependent variable (they either pay bribes or they do not). Because the coefficients from probit regressions are relatively meaningless on their own, I also ran separate regressions to estimate the marginal effect of each independent variable on bribe paying. This marginal effect coefficient will show the change in the conditional probability that firms will pay bribes if they fall within a particular group (change in Pr(BD=1 — var=1)). The probit regression coefficient results are included in the Appendix (Appendix Table 2) and the marginal effect coefficient results are shown below in Table 2.

TABLE 1
Summary Statistics of variables considered in the model

Variable	Mean	Std. Dev.	Min	Max
A. Firm Level Variables				
Bribe (Br)	0.363	0.481	0	1
Government Help (GH)	0.475	0.499	0	1
Access to Credit (AC)	0.133	0.340	0	1
Credit Constrained (CC)	0.649	0.477	0	1
Registered (R)	0.541	0.498	0	1
Foreign (F)	0.167	0.373	0	1
Trust in Courts (TC)	0.437	0.496	0	1
Competition (C)	0.524	0.499	0	1
Government Ownership (GO)	0.016	0.126	0	1
Medium (M)	0.266	0.442	0	1
Large (L)	0.096	0.294	0	1
Young (Y)	0.737	0.440	0	1
Old (O)	0.032	0.177	0	1
Sales (S)	17.231	2.782	6.644	34.105
Trade (T)	0.180	0.385	0	1
B. Macro Level Variables				
IndoChina Trade (IC)	0.577	0.494	0	1
IndoChina x Foreign (ICF)	0.076	0.265	0	1
British Legal Origin (BLO)	0.480	0.500	0	1

VI. Results and Discussion

As Table 2 clearly illustrates, the majority of the variables selected for the regression demonstrate a significant relationship with a firm's decision to engage in bribe paying. Beginning with the three hypotheses - Control Rights, Bargaining Power, and Grease the Wheels - I find evidence in support of each theory. Looking at the marginal effects presented in Table 2, we find that ceteris paribus, Government Help increased the probability of bribe paying by 12%, Access to Credit increased the probability by 4%, Credit Constrained increased the probability by 2%, Trust in Courts reduced the probability by 10%, and Competition increased the probability by 7% (contrary to the belief that competition drives out corruption, bribe paying might lead to a needed competitive advantage). Each of these marginal effects and the original coefficients from the probit regressions (See Appendix Table 2) carries the expected sign and shows that bribe paying in Africa is indeed a function of the institutional investment climate as much as it is a supply-

side management decision.

Table 2
Marginal Effects from the dprobit regression [Dependent Variable = BD]

	Model 1	Model 2	Model 3	Model 4	Model 5
Government Help	**0.122*** **	**0.122*** **	**0.122*** **	**0.122*** **	**0.122*** **
	(0.008)	(0.008)	(0.008)	(0.008)	(0.008)
Access to Credit	**0.043*** **	**0.043*** **	**0.043*** **	**0.043*** **	**0.043*** **
	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)
Credit Constrained	**0.021** **	**0.021** **	**0.021** **	**0.021** **	**0.021** **
	(0.009)	(0.009)	(0.009)	(0.009)	(0.009)
Trust in Courts	**-0.102*** **	**-0.102*** **	**-0.102*** **	**-0.102*** **	**-0.102*** **
	(0.008)	(0.008)	(0.008)	(0.008)	(0.008)
Competition	**0.070*** **	**0.070*** **	**0.070*** **	**0.070*** **	**0.070*** **
	(0.009)	(0.009)	(0.009)	(0.009)	(0.009)
Foreign	-0.012	-0.012	-0.016	-0.016	-0.012
	(0.012)	(0.012)	(0.016)	(0.016)	(0.012)
Registered	**-0.045*** **	**-0.045*** **	**-0.045*** **	**-0.045*** **	**-0.045*** **
	(0.012)	(0.012)	(0.012)	(0.012)	(0.012)
Government Owned	**-0.068** **	**-0.068** **	**-0.067** **	**-0.067** **	**-0.068** **
	(0.032)	(0.032)	(0.032)	(0.032)	(0.032)
Medium	**0.026** **	**0.026** **	**0.026** **	**0.026** **	**0.026** **
	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)
Large	**-0.036** **	**-0.036** **	**-0.036** **	**-0.036** **	**-0.036** **
	(0.017)	(0.017)	(0.017)	(0.017)	(0.017)
Young	0.015	0.015	0.015	0.015	0.015
	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)
Old	**-0.046***	**-0.046***	**-0.046***	**-0.046***	**-0.046***
	(0.024)	(0.024)	(0.024)	(0.024)	(0.024)
Sales	**-0.006*** **	**-0.006*** **	**-0.006*** **	**-0.006*** **	**-0.006*** **
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Trade	**0.039*** **	**0.039*** **	**0.039*** **	**0.039*** **	**0.039*** **
	(0.012)	(0.012)	(0.012)	(0.012)	(0.012)
IndoChina		**-0.319*** **		**-0.321*** **	
		(0.087)		(0.087)	
IndoChina x Foreign			0.008	0.008	
			(0.023)	(0.023)	
British Legal Origin					-0.111
					-0.086
Observations	18101	18101	18101	18101	18101
Pseudo R^2	.199	.199	.199	.199	.199

*, **, *** indicate significance at the 10%, 5%, and 1% level respectively.
Every model includes a full set of country, industry, and year dummies

Importantly, the Foreign dummy does not have significance in any of the models, but if it did, it would have a negative marginal effect on bribe paying. This might prove the effectiveness of Western led anti-corruption legislation, or could also be attributed to other explanations such as greater bargaining power held by foreign firms or less knowledge of domestic business practices where bribery is in fact the norm.

With the exception of the Young dummy variable, each of the other firm level control variables also showed robust sig-

nificance. Registered firms, Government Owned firms, Large firms, Old firms, and firms with more sales all proved less likely to engage in bribe paying. While Government Owned firms might have less reporting of bribe incidence, this trend among the other groups is most likely due to the better bargaining position that these firms have against public officials requesting bribes. As more established entities with greater resources, they are more empowered to report and seek legal action against corrupt public officials.

Moving on to the macro level variables, I find an extremely interesting result. Firms operating in countries with high levels of Indian and Chinese investment and trade activity are 32% less likely to engage in bribery. That number is remarkable, considering that Government Help demonstrated the next highest marginal effect at 10%. Of course, this result does not necessarily indicate that Indian and Chinese firms contribute to a less corrupt business environment. A far more likely explanation is that countries that attract FDI and trade have inherently better investment climates that are already relatively corruption free. However, some of the countries included in the IndoChina group such as the DRC, Nigeria, and Sudan (to name a few) are hardly known to for their investor friendly environments. To check these results, Model 5 substitutes the British Legal Origin Dummy (originally excluded because it is highly correlated with IndoChina) and finds that it has a much smaller marginal effect and is not significant at even the 10% level. This result is interesting because the BL dummy should serve as a good proxy of investment climate and rule of law, and even though it is highly correlated with IndoChina, it does not bear the same result.

Models 2 and 3 include the IndoChina x Foreign interaction term, and although the marginal effect is slightly positive it is not significant in either specification. This term is admittedly a very rough attempt to disaggregate the effect of Indian and Chinese foreign firms more specifically, and the results are therefore unsurprisingly inconclusive. This could be for any number of reasons including a lack of specificity regarding

country of origin. However, because the Foreign dummy also lacks a significant relationship with corruption throughout the continent, we cannot discount the possibility that foreign firms simply adapt to the most common business practices (corrupt or not) in the host country.

VII. Conclusion and Policy Implications

The rise of India and China as global powers has changed the status quo in the private sectors of several key African economies. The Western response has been acute and critical; Secretary Clinton's stark words in 2012 leave no confusion surrounding the West's protectionist - not to be confused for altruistic - attitude. However, despite the West's preferences, the sheer volume and trends in Indian and Chinese trade and investment in the continent show that they will remain major actors in the region for the foreseeable future. A 2014 report by McKinsey & Company predicts that Africa will become the fastest growing region in the next few decades and shows that Indian and Chinese involvement will be instrumental in leading that growth.

That said, Western concerns that imported Indian and Chinese management techniques will weaken governance are not entirely off the mark. Anecdotal evidence presented in this paper shows that even the "cleanest" Indian and Chinese firms have been caught up in corruption allegations at home, if not in Africa. Although these cases are far more common for large Chinese state owned enterprises with fewer accountability and transparency checks, Indian firms have had their own issues in recent years. However, it is important to remember that despite Western anti-corruption legislation, Western firms have also been found guilty of their own share of questionable deals in Africa. Legislation or not, bribe paying (and taking) remains a part of the business environment in virtually every African country. The empirical results support the idea that corruption is primarily driven by demand-side institutional factors that are met by supply-side firm practices.

The surprising finding that firms in countries with heavy

Indian and Chinese involvement are 30% less likely to pay a bribe is, however, promising preliminary evidence that these new actors are not worsening the situation. Contrary to Western claims of a "race to the bottom" scenario, this might actually indicate an opposite trend in which an increased supply of investment funds gives African governments more bargaining power to strike better deals with more responsible corporate actors. Understandably, this process also creates greater opportunities for bribe taking, and anti-corruption efforts will need to continue at the country level.

Notably, these results include the caveat that the Enterprise Surveys do not include data from firms operating in extractive industries, where much of the criticism regarding the corrupt practices of Indian and Chinese firms has been focused. Functionally, this means that the findings of this paper could be skewed toward a more favorable representation of Indian and Chinese involvement on the African continent. However, as I note in the literature review, firms operating in these extractive industries have increasingly become the minority among Indian and Chinese companies interested in doing business in Africa. As business opportunities become more widespread in a larger variety of industries, the trends highlighted by this paper will likely become even more relevant.

Additionally, efforts to promote strong corporate governance should be undertaken at the firm level in order to solve the collective action problem of ending corruption in the private sector. Because refusing to pay a bribe can put a firm at a disadvantage to a direct competitor, it is difficult to convince its managers to maintain their anti-corruption position. Coordinated efforts like the UN Global Compact (which includes several Indian and Chinese firms) and the IFC's Africa Corporate Governance Network represent promising first steps and should continue to be supported (United Nations, 2010; IFC, 2013).

As the Indian and Chinese presence in Africa continues to grow, there is room for further research regarding its impact on governance and corruption. The World Bank Enterprise

Surveys have provided a good starting point, but a more detailed firm level dataset will be required to draw convincing conclusions in one direction or the other. Too much emphasis has been placed on the new East-West rivalry in Africa and the discussion has been colored by platitudes rather than by hard empirical evidence. However, as this data becomes increasingly available, firms - whether Indian, Chinese, European, or American - and African government officials alike will hopefully be held more accountable for positive governance outcomes.

Appendix

Table 1: Correlation matrix of all independent variables considered for the empirical model specifications

	Government Help (GH)	Access to Credit (AC)	Credit Constrained (CC)	Registered (R)	Foreign (F)	Trust in Courts (TC)	Competition (c)	Government Ownership (GO)	Medium (M)
Government Help (GH)	1.000								
Access to Credit (AC)	0.128	1.000							
Credit Constrained (CC)	0.013	-0.057	1.000						
Registered (R)	-0.067	0.114	-0.054	1.000					
Foreign (F)	0.051	0.088	-0.127	0.097	1.000				
Trust in Courts (TC)	-0.060	0.018	-0.093	0.033	0.001	1.000			
Competition (c)	0.021	-0.031	0.114	0.253	-0.060	-0.049	1.000		
Government Ownership (GO)	0.023	0.049	-0.003	0.041	0.114	-0.004	-0.013	1.000	
Medium (M)	0.055	0.138	-0.109	0.114	0.104	0.009	-0.030	0.018	1.000
Large (L)	0.092	0.217	-0.105	0.135	0.205	0.022	-0.078	0.122	-0.145
Young (Y)	-0.007	-0.119	0.049	-0.083	-0.080	0.009	0.001	-0.085	-0.134
Old (O)	0.014	0.085	-0.036	0.079	0.085	0.003	-0.008	0.068	0.046
Sales (S)	0.096	0.183	-0.075	0.170	0.224	-0.061	0.045	0.096	0.199
Trade (T)	0.282	0.219	-0.086	0.087	0.205	-0.019	-0.041	0.077	0.120
IndoChina Trade (ICT)	-0.017	-0.060	-0.028	-0.055	-0.108	-0.018	0.019	-0.034	0.024
IndoChina x Foreign (ICF)	0.002	0.038	-0.092	0.098	0.632	-0.030	-0.030	0.043	0.092
British Legal Origin (BLO)	-0.006	-0.004	-0.065	-0.078	-0.104	0.102	0.007	-0.018	0.049

	Large (L)	Young (Y)	Old (O)	Sales (S)	Trade (T)	IndoChina Trade (ICT)	IndoChina x Foreign (ICF)	British Legal Origin (BLO)
Large (L)	1.000							
Young (Y)	-0.205	1.000						
Old (O)	0.199	-0.309	1.000					
Sales (S)	0.318	-0.112	0.078	1.000				
Trade (T)	0.271	-0.120	0.111	0.240	1.000			
IndoChina Trade (ICT)	0.005	0.019	-0.001	0.076	-0.033	1.000		
IndoChina x Foreign (ICF)	0.152	-0.055	0.074	0.255	0.150	0.246	1.000	
British Legal Origin (BLO)	0.055	-0.071	0.072	0.020	-0.007	0.510	0.080	1.000

Table 2
Parameter estimates from the probit regression [Dependent Variable = BD]

	Model 1	Model 2	Model 3	Model 4	Model 5
Government Help (GH)	**0.326*** **	**0.326*** **	**0.326*** **	**0.326*** **	**0.326*** **
	(0.023)	(0.023)	(0.023)	(0.023)	(0.023)
Access to Credit	**0.112*** **	**0.112*** **	**0.112*** **	**0.112*** **	**0.112*** **
	(0.034)	(0.034)	(0.034)	(0.034)	(0.034)
Credit Constrained	**0.056** **	**0.056** **	**0.056** **	**0.056** **	**0.056** **
	(0.024)	(0.024)	(0.024)	(0.024)	(0.024)
Trust in Courts	**-0.275*** **	**-0.275*** **	**-0.275*** **	**-0.275*** **	**-0.275*** **
	(0.022)	(0.022)	(0.022)	(0.022)	(0.022)
Competition	**0.189*** **	**0.189*** **	**0.189*** **	**0.189*** **	**0.189*** **
	(0.025)	(0.025)	(0.025)	(0.025)	(0.025)
Foreign	-0.033	-0.033	-0.042	-0.042	-0.033
	(0.032)	(0.032)	(0.042)	(0.042)	(0.032)
Registered	**-0.120*** **	**-0.120*** **	**-0.120*** **	**-0.120*** **	**-0.120*** **
	(0.033)	(0.033)	(0.033)	(0.033)	(0.033)
Government Owned	**-0.188** **	**-0.188** **	**-0.187** **	**-0.187** **	**-0.188** **
	(0.092)	(0.092)	(0.092)	(0.092)	(0.092)
Medium	**0.070** **	**0.070** **	**0.070** **	**0.070** **	**0.070** **
	(0.028)	(0.028)	(0.028)	(0.028)	(0.028)
Large	**-0.097** **	**-0.097** **	**-0.097** **	**-0.097** **	**-0.097** **
	(0.048)	(0.048)	(0.048)	(0.048)	(0.048)
Young	0.041	0.041	0.041	0.041	0.041
	(0.027)	(0.027)	(0.027)	(0.027)	(0.027)
Old	**-0.125***	**-0.125***	**-0.126***	**-0.126***	**-0.125***
	(0.067)	(0.067)	(0.067)	(0.067)	(0.067)
Sales	**-0.017*** **	**-0.017*** **	**-0.017*** **	**-0.017*** **	**-0.017*** **
	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)
Trade	**0.103*** **	**0.103*** **	**0.103*** **	**0.103*** **	**0.103*** **
	(0.031)	(0.031)	(0.031)	(0.031)	(0.031)
Indochina		**-0.305*** **		**-0.310*** **	
		(0.090)		(0.091)	
Indochina x Foreign			0.021	0.021	
			(0.061)	(0.061)	
British Legal Origin					-0.298
					-0.233
Observations	18101	18101	18101	18101	18101
Pseudo R^2	.199	.199	.199	.199	.199

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Bayesian Portfolio Analysis: Analyzing the Global Investment Market

Daniel Roeder[1]

Abstract

The goal of portfolio optimization is to determine the ideal allocation of assets to a given set of possible investments. Many optimization models use classical statistical methods, which do not fully account for estimation risk in historical returns or the stochastic nature of future returns. By using a fully Bayesian analysis, however, I am able to account for these aspects and incorporate a complete information set as a basis for the investment decision. I use Bayesian methods to combine different estimators into a succinct portfolio optimization model that takes into account an investor's utility function. I will test the model using monthly return data on stock indices from Australia, Canada, France, Germany, Japan, the U.K. and the U.S.

I. Introduction

Portfolio optimization is one of the fastest growing areas of research in financial econometrics, and only recently has computing power reached a level where analysis on numerous assets is even possible. There are a number of portfolio optimization models used in financial econometrics and many of them build on aspects of previously defined models. The model I will be building uses Bayesian statistical methods to combine insights from Markowitz, BL and Zhou. Each of these papers use techniques from the previous one to specify and create a novel modeling technique.

Bayesian statistics specify a few types of functions that are necessary to complete an analysis, the prior distribution, the

[1]I am an undergraduate senior at Duke University double majoring in Economics and Statistics. I would like to thank both Scott Schmidler and Andrew Patton for serving as my advisors on this thesis. I would also like to thank my parents, Sandra Eller and Greg Roeder, for their love, guidance and support throughout my life.

likelihood function, and the posterior distribution. A prior distribution defines how one expects a variable to be distributed before viewing the data. Prior distributions can be of different weights in the posterior distribution depending on how confident one is in their prior. A likelihood function describes the observed data in the study. Finally, the posterior distribution describes the final result, which is the combination of the prior distribution with the likelihood function. This is done by using Bayes theorem [2], which multiplies the prior times the posterior and divides by the normalizing constant, which conditions that the probability density function (PDF) of the posterior sums to 1. Bayesian analysis is an ideal method to use in a portfolio optimization problem because it accounts for the estimation risk in the data. The returns of the assets form a distribution centered on the mean returns, but we are not sure that this mean is necessarily the true mean. Therefore it is necessary to model the returns as a distribution to account for the inherent uncertainty in the mean, and this is exactly what Bayesian analysis does.

Zhou incorporates all of the necessary Bayesian components in his model; the market equilibrium and the investor?s views act as a joint prior and the historical data defines the likelihood function. This strengthens the model by making it mostly consistent with Bayesian principles, but some aspects are still not statistically sound. In particular, I disagree with the fact that Zhou uses the historical covariance matrix, Σ, in each stage of the analysis (prior and likelihood). The true covariance matrix is never observable to an investor, meaning there is inherent uncertainty in modeling Σ, which must be accounted for in the model. Zhou underestimates this uncertainty by using the historical covariance matrix to initially estimate the matrix, and by re-updating the matrix with the historical data again in the likelihood stage. This method puts too much confidence in the historical matrix by re-updating the prior with the same historical matrix. I plan to account for this uncertainty by incorporating an inverse-Wishart prior

[2] $P(\Theta|Y) = \frac{P(Y|\Theta)P(\Theta)}{\int P(Y|\Theta)P(\Theta)\,\mathrm{d}\Theta}$

distribution on the Black-Litterman prior estimate, which will model Σ as a distribution and not a point estimate. The inverse-Wishart prior will use the Black-Litterman covariance matrix as a starting point, but the investor can now model the matrix as a distribution and adjust confidence in the starting point with a tuning parameter. This is a calculation that must be incorporated to make the model statistically sound, and it also serves as a starting point for more extensive analysis of the covariance matrix.

The empirical analysis in Zhou is based on equity index returns from Australia, Canada, France, Germany, Japan, the United Kingdom and the United States. My dataset is comprised of the total return indices for the same countries, but the data spans through 2013 instead of 2007 like in Zhou. This is a similar dataset to that chosen by BL, which was used in order to analyze different international trading strategies based on equities, bonds and currencies.

The goal of this paper is to extend the Bayesian model created by Zhou by relaxing his strict assumption on the modeling of the covariance matrix by incorporating the inverse-Wishart prior extension. This will in turn create a statistically sound and flexible model, usable by any type of investor. I will then test the models by using an iterative out-of-sample modeling procedure.

In section II, I further describe the literature on the topic and show how it influenced my analysis. In section III I will describe the baseline models and the inverse-Wishart prior extension. In Section IV I will summarize the dataset and provide descriptive statistics. In section V I will describe how the models are implemented and tested. In Section VI I will describe the results and compare the models, and in Section VII I will offer conclusions and possible extensions to my model.

II. Literature Review

Models

Harry Markowitz established one of the first frameworks for portfolio optimization in 1952. In his paper, Portfolio Selection, Markowitz solves for the portfolio weights that maximize a portfolio?s return while minimizing the volatility, by maximizing a specified expected utility function for the investor. The utility function is conditional on the historical mean and variance of the data, which is why it is often referred to as a mean-variance analysis. These variables are the only inputs, so the model tends to be extremely sensitive to small changes in either of them. The model also assumes historical returns on their own predict future returns, which is something known to be untrue in financial econometrics.

These difficulties with the mean-variance model do not render it useless. In fact, the model can perform quite well when there are better predictors for the expected returns and covariance matrix (rather than just historical values). The model by BL extends the mean-variance framework by creating an estimation strategy that incorporates an investor?s views on the assets in question with an equilibrium model of asset performance. Many investors make decisions about their portfolio based on how they expect the market to perform, so it is intuitive to incorporate these views into the model.

Investor views in the Black-Litterman model can either be absolute or relative. Absolute views specify the expected return for an individual security; for example, an investor may think that the S&P 500 will return 2% next month. Relative views specify the relationship between assets; for example, an investor may think that the London Stock Exchange will have a return 2% higher than the Toronto Stock Exchange next month. BL specify the same assumptions and use a similar model to Markowitz to describe the market equilibrium, and they then incorporate the investor?s views through Bayesian updating. This returns a vector of expected returns that is similar to the market equilibrium but adjusted for the in-

vestor?s views. Only assets that the investor has a view on will deviate from the equilibrium weight. Finally, BL use the same mean-variance utility function as Markowitz to calculate the optimal portfolio weights based off of the updated expected returns.

Zhou takes this framework one step further by also incorporating historical returns into the analysis because the equilibrium market weights are subject to error that the historical data can help fix. The market equilibrium values are based on the validity of the capital asset pricing model (CAPM)[3], which is not always supported by historical data. This does not render the equilibrium returns useless; they simply must be supplemented by historical data in order to make the model more robust. The combination of the equilibrium pricing model and the investor?s views with the data strengthens the model by combining different means of prediction. As an extension, it would be useful to research the benefit of including a more complex data modeling mechanism that incorporates more than just the historical mean returns. A return forecasting model could be of great use here, though it would greatly increase the complexity of the model.

Zhou uses a very complete description of the market by incorporating all three of these elements, but there is one other aspect of the model that he neglects; his theoretical framework does not account for uncertainty in the covariance matrix. By neglecting this aspect, he implies that the next period's covariance matrix is only described by the fixed historical covariance matrix. This is in line with the problems that arise in Markowitz, and is also not sound in a Bayesian statistical sense because he is using a data generated covariance matrix in the prior, which is then updated by the same data. I will therefore put an inverse-Wishart prior distribution on the Black-Litterman estimate of Σ before updating the prior with the data. The primary Bayesian updating stage, where the equilibrium estimate is updated by the investor views will re-

[3]For more information regarding the choice of the market equilibrium model, see BL(1992)

main consistent. This way Σ is modeled as a distribution in the final Bayesian updating stage which will allow the prior to have a more profound effect.

Investment Strategies

Though the Black-Litterman model is quantitatively based it is extremely flexible, unlike many other models, due to the input of subjective views by the investor. These views are directly specified and can come from any source, whether that is a hunch, the Wall Street Journal, or maybe even an entirely different quantitative model. I will present a momentum based view strategy, but this is only one of countless different strategies that could be incorporated, whether they are quantitatively based or not. The results of this paper will be heavily dependent on the view specification, which is based on the nature of the model. The goal of this paper is not to have a perfect empirical analysis, but instead to present a flexible, statistically sound and customizable model for an investor regardless of their level of expertise.

The investor's views can be independent over time or follow a specific investment strategy. In the analysis I use a function based on the recent price movement of the indices, a momentum strategy, to specify the views. The conventional wisdom of many investors is that individual prices and their movements have nothing to say about the asset's value, but when the correct time frame is analyzed, generally the previous 6-12 months, statistically significant returns can be achieved (Momentum). In the last 5 years alone, over 150 papers have been published investigating the significance of momentum investment strategies (Momentum). Foreign indices are not an exception, as it has been shown that indices with positive momentum perform better than those with negative momentum (AQR).

The basis of momentum strategies lies in the empirical failure of the efficient market hypothesis, which states that all possible information about an asset is immediately priced into the asset once the information becomes available. This tends

to fail because some investors get the information earlier or respond to it in different manners, so there is an inherent asymmetric incorporation of information that creates short-term price trends (momentum) that can be exposed. This phenomenon can be further explored in Momentum.

Though momentum investing is gaining in popularity, there are countless other investment strategies in use today. Value and growth investing are both examples, and view functions incorporating these strategies are an interesting topic of further research.

III. Theoretical Framework

Baseline

As mentioned in the literature review, Markowitz specifies a mean-variance utility function with respect to the portfolio asset weight vector, w. The investor's goal is to maximize the expected return while minimizing the volatility and he does so by maximizing the utility function

$$U(w) = E[R_{t+1}] - \frac{\gamma}{2} Var[R_{t+1}] = w'\mu - \frac{\gamma}{2} w'\Sigma w, \qquad (1)$$

where R_T is the current period?s return, R_{T+1} is the future period?s return, γ is the investor?s risk aversion coefficient, μ is the sample return vector and Σ is the sample covariance matrix. This is referred to as a two moment utility function since it incorporates the distribution's first two moments, the mean and variance. The first order condition of this utility function, with respect to w, solves to

$$w = \frac{1}{\gamma} \Sigma^{-1} \mu, \qquad (2)$$

which can be used to solve for the optimal portfolio weights given the historical data.

BL first specify their model by determining the expected market equilibrium returns. To do so, they solve for μ in (2) by

plugging in the sample covariance matrix and the market equilibrium weights. The sample covariance matrix comes from the data and the market equilibrium weights are simply the percentage that each country's market capitalization makes up of the total portfolio market capitalization.

In equilibrium, if we assume that the CAPM holds and that all investors have the same risk aversion and views on the market, the demand for any asset will be equal to the available supply. The supply of an asset is simply its market capitalization, or the amount of dollars available of the asset in the market. In equilibrium when supply equals demand, we know that the weights of each asset in the optimal portfolio will be equal to the supply, or the market capitalization of each asset. Σ is simply the historical covariance matrix, so we therefore know both w and Σ in (2), meaning we can solve for μ^e, the equilibrium expected excess returns.

It is also assumed that the true expected excess return, μ, is normally distributed with mean μ^e and covariance matrix $\tau\Sigma$. This can be written as

$$\mu = \mu^e + \epsilon^e, \quad \epsilon \sim N(0, \tau\Sigma) \tag{3}$$

where μ^e is the market equilibrium returns, τ is a scalar indicating the confidence of how the true expected returns are modeled by the market equilibrium, and Σ is the fixed sample covariance matrix. It is common practice to use a small value of tau since one would guess that long-term equilibrium returns are less volatile than historical returns.

We must also incorporate the investor?s views, which can be modeled by

$$P\mu = \mu^v + \epsilon^v, \quad \epsilon \sim N(0, \Omega), \tag{4}$$

where P is a K×N matrix that specifies K views on the N assets, and Ω is the covariance matrix explaining the degree of confidence that the investor has in his views. Ω is one of the harder variables to specify in the model, but [**?**] provide a method that also helps with the specification of τ. Ω is a diagonal matrix since it is assumed that views are independent of

one another, meaning all covariance (non-diagonal) elements of the matrix are zero. Each diagonal element of Ω can be thought of as the variance of the error term, which can be specified as $P_i \Sigma P_i'$, where P_i is an individual row (view) from the K $\times$ N view specifying matrix, and Σ is again the historical covariance matrix. Again, I do not agree with this overemphasis on the historical covariance matrix, but I include it here for simplicity of explaining the intuition of the model.

Intuition calibrate the confidence of each view by shrinking each view's error team by multiplying it by τ. This makes τ independent of the posterior analysis because it is now incorporated in the same manner in the two stages of the model. If it is drastically increased, so too are be the error terms of Ω, but the estimated return vector, shown in (5) is not changed because there is be an identical effect on Σ.

We can combine these two models by Bayesian updating, which leaves us with the Black-Litterman mean and variance

$$\mu_{BL} = [(\tau\Sigma)^{-1} + P'\Omega P]^{-1}[(\tau\Sigma^{-1}\mu^e + P'\Omega^{-1}\mu^v] \qquad (5)$$

$$\Sigma_{BL} = \Sigma + [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}. \qquad (6)$$

The Black-Litterman posterior covariance matrix is simply $[(\tau\Sigma_h)^{-1}+P'\Omega^{-1}P]^{-1}$. The extra addition of Σ occurs because the investor must account for the added uncertainty of making a future prediction. This final distribution is referred to as the posterior predictive distribution and is derived through Bayesian updating. There is an added uncertainty in making a prediction of an unknown, future value, and to account for this the addition of Σ is necessary.

It is assumed that both the market equilibrium and the investor?s views follow a multivariate normal distribution, so it is known that the posterior predictive distribution is also multivariate normal due to conjugacy. In order to find the optimal portfolio weights μ_{BL} and Σ_{BL} are simply plugged into (2).

Once the Black-Litterman results are specified we have the joint prior for the Bayesian extension. We combine this prior with the normal likelihood function describing the data [4], and based off of Bayesian updating logic we obtain the posterior predictive mean, μ_{bayes} and covariance matrix, Σ_{bayes},

$$\mu_{bayes} = [\Delta^{-1} + (\Sigma/T)^{-1}]^{-1}[\Delta^{-1}\mu_{bl} + (\Sigma/T)^{-1}\mu_h] \tag{7}$$

$$\Sigma_{bayes} = \Sigma + [(\Delta^{-1} + (\Sigma/T)^{-1}]^{-1} \tag{8}$$

where Σ is the historical covariance matrix, μ_h are the historical means of the asset returns, $\Delta = (\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}$ is the covariance matrix of the Black-Litterman estimate, and T is the sample size of the data, which is the weight prescribed to the sample data. The larger the sample size chosen, the larger the weight the data has in the results. It is common practice to let T = n, unless we do not have a high level of confidence in the data and want T <n. The number of returns is specified independently from the data because only the sample mean and covariance matrix are used in the analysis, not the individual returns. This is ideal because it allows the investor to set the confidence in the data without the sample size doing it automatically. Historical return data is often lengthy, but that does not necessarily mean a high degree of confidence should be prescribed to it.

Analogous to the Black-Litterman model, the posterior estimate of Σ in Zhou is $[(\Delta^{-1} + (\Sigma/T)^{-1}]^{-1}$. The addition of Σ to the posterior in calculating Σ_{bayes} is necessary to account for the added uncertainty of the posterior predictive distribution. The theory behind this is identical to that in the Black-Litterman model.

It is known that both the prior and likelihood follow a multivariate normal distribution, so due to conjugacy the same is true of the posterior predictive distribution. The posterior

[4] Zhou (2009) makes the same assumptions on returns as BL, that they are i.i.d.

mean is a weighted average of the Black-Litterman returns and the historical means of the asset returns. As the sample size increases, so does the weight of the historical returns in the posterior mean. In the limit if T = ∞, then the portfolio weights are identical to the mean-variance weights, and if T = 0 then the weights are identical to the Black-Litterman weights.

Extension

As it stands, the Zhou model uses the sample covariance matrix in the prior generating stage, even though in a fully Bayesian analysis a full incorporation of historical data is not supposed to occur outside of the likelihood function. This means the data is used to generate the prior views, and then further update the views by again incorporating the data through the likelihood function.

To account for the uncertainty of modeling Σ under the historical covariance matrix in each stage, I will impose an inverse-Wishart prior on the Black-Litterman covariance matrix. Under this method, the historical covariance matrix will still be used in both Bayesian updating stages, but I can now better account for the potential problems of doing so through the inverse-Wishart prior.

The inverse-Wishart prior changes only the specification of Σ , not μ, and is specified by $\mathcal{W}_{-1}(\Psi, v.0)$ where Ψ is the prior mean of the covariance matrix, and $v.0$ is the degrees of freedom of the distribution. The larger the degrees of freedom, the more confidence the investor has in Ψ as an estimate of Σ. In this case, $\Psi = \Sigma_{BL}$, and $v.0$ can be thought of as the number of "observations" that went into the prior[5].

The prior is then updated by the likelihood function, the historical estimate of Σ. μ_{BL} is also updated by the historical data, but the analysis does not change the specification of μ since the prior is only put on the Σ_{BL}. The pos-

[5]Though no actual historical data observations were used in forming the prior, this interpretation keeps the model consistent given how the Bayesian updating process is conducted

terior distribution of Σ is also an inverse-Wishart distribution due to the conjugate Bayesian update and is defined as $\mathcal{W}_{-1}((\Psi+S_\mu), (v.0+T))$, where S_μ is the historical data generated sum of squares matrix, and T the number of observations that were used to form the likelihood. T is specified in the same manner as in the Zhou model; it is up to the investor to set confidence in the data through T as it does not necessarily need to be the actual number of observations.

I use the mean of the posterior inverse-Wishart distribution to define the posterior covariance matrix of the extension. The mean of the posterior is defined as $E[\Sigma \mid \mu, y_1, ..., y_n] = \frac{1}{v.0+T-n-1}(\Psi + S_\mu)$, where $y_1, ..., y_n$ is the observed data, and n is the number of potential assets in the portfolio. This posterior matrix is then added to the historical covariance matrix in order to get the posterior predictive value, Σ_{ext}. The specification of μ is not affected under this model so $\mu_{ext} = \mu_{bayes}$

IV. Data

Monthly dollar returns from 1970-2013, for the countries in question [6] were obtained from Global Financial Data, and I used that raw data to calculate the $n = 528$ monthly percent returns. The analysis is based on excess returns, so assuming the investor is from the U.S. I use the 3-month U.S. Treasury bill return as the risk-free rate.

Data must also be incorporated to describe the market equilibrium state of the portfolio. I collected this data from Global Financial Data and am using the market capitalizations of the entire stock markets in each country from January, 1980 to December, 2013. Given the rolling window used in my analysis, January, 1980 is the first month where market equilibrium data is needed

Table 1 presents descriptive statistics for the seven country indices I am analyzing. The mean annualized monthly excess returns are all close to seven percent and the standard deviations are all close to 20 percent. The standard deviation for

[6] Australia, Canada, France, Germany, Japan, the U.K. and the U.S.

the U.S. is much smaller than the other countries, which makes sense because safer investments generally have less volatility in returns. All countries exhibit relatively low skewness, and most countries have a kurtosis that is not much larger than the normal distributions kurtosis of 3. The U.K. deviates the most from the normality assumption given it has the largest absolute value of skewness and a kurtosis that is almost two times as large as the next largest kurtosis. I am not particularly concerned by these values, however, because the dataset is large and the countries do not drastically differ from a normal distribution. The U.K. is the most concerning, but a very large kurtosis is less problematic than a very large skewness and the skewness is greatly influenced by one particularly large observation that occurred in January of 1975, during a recession. Though the observation is an outlier, it seems to have occurred under legitimate circumstances so I include it in the analysis.

Table 1: Analysis of Country Index Returns

Country	Mean (%)	St. Dev. (%)	Skewness (%)	Kurtosis
Australia	7.86	23.68	-0.84	7.54
Canada	5.91	19.44	-0.62	5.57
France	7.43	22.83	-0.18	4.39
Germany	6.83	21.30	-0.37	4.46
Japan	6.11	21.04	0.23	3.79
UK	7.97	22.38	0.98	13.60
US	6.15	15.49	-0.45	4.78

V. Model Implementation

Rolling Window

A predictive model is best tested under repeated conditions when it uses a subset of the data as "in-sample" data to predict the "out-of sample" returns. This simulates how a model would be implemented in a real investment setting since there is obviously no data incorporated in the model for the future prediction period. If I were to include the observations I was also trying to predict, I would artificially be increasing the

predictive power of the model by predicting inputs.

I am using a 10 year rolling window as the in sample data to predict the following month. I begin with the first 10 years of the dataset, January, 1970 - December, 1980, to predict returns and optimal asset weights for the following month, January 1981. I then slide the window over one month and use February, 1970 - January, 1981 to predict returns and optimal asset allocations for February, 1981. The dataset spans through 2013, giving me 528 individual returns. I therefore calculate 408 expected returns and optimal weights.

It is quite easy to assess performance once each set of optimal weights is calculated since there is data on each realized return. For each iteration I calculate the realized return for the entire portfolio by multiplying each individual index's weight by its corresponding realized return. I do not have any investment constraints in the model so I also need to account for the amount invested in, or borrowed from, the risk-free rate. One minus the sum of the portfolio weights is the amount invested in (or borrowed from, if negative) the risk-free rate.

Momentum Based views

In order to be able to run the model in an updating fashion, I must to create a function that will iteratively specify the investor's views, and I will do so using a momentum based investment strategy. I have created a function that uses both a primary absolute strategy and a secondary relative strategy that is explained below.

The primary strategy estimates absolute views based on the mean and variance of the previous twelve months, since this is the known window for Momentum. This is a loose adaption of our momentum strategy that specifies that stocks that have performed well in the past twelve months will continue to do so in the following month. By taking the mean I can account for the fact that at many times, the indices have no momentum, in which case I expect the mean to be close to zero. For this strategy, since I am only specifying absolute views, the P matrix is an identity matrix with a dimension

equal to the number of assets in question. The Omega matrix is again calculated using the method specified by Intuition.

The secondary strategy, which is appended to both of these primary strategies, if the conditions hold, attempts to find indices that are gaining momentum quickly in the short term. To do this I look at the last 4 months of the returns to see if they are consistently increasing or decreasing. If the index is increasing over the four months, it is given a positive weight, and if it is decreasing over the four months it is given a negative weight. I use a four-month increasing scheme to catch the indices under momentum before they hit the standard six-month cutoff. The weights are determined by a method similar to the market capitalization weighting method used by Idzorek. The over-performing assets are weighted by the ratio of the individual market capitalization to the total over-performing market capitalization, and the same goes for under-performing assets. This puts more weight on large indices, which is intuitive because there is likely more potential for realized returns in this case. The expected return of this view is a market capitalization weighted mean of each of the indices that have the specified momentum.

This is a fairly strict strategy, which is why I refer to it as secondary. For each iteration, sometimes there are no underperforming or over performing assets under the specifications. In this case, only the primary strategy is used. If assets do appear to have momentum given the definition, then it is appended to the P matrix along with the primary strategy.

VI. Results

The results of the four models are presented below in Table 2. It must be considered that the results are heavily dependent on the dataset and the view specifying function, two aspects of the model that are not necessarily generalizable to an investor. Further empirical analysis of the models is therefore necessary to determine which is best under the varying conditions of the current investment market.

Table 2: Portfolio Optimization Results

Model	Mean (%)	Volatility (%)	Skewness (%)	Kurtosis
Markowitz	2.82	39.94	-.39	4.46
Black-Litterman	7.24	15.02	0.51	4.19
Zhou	4.62	27.77	-.47	4.70
Extension	4.75	9.52	-.47	4.67

Note: The values in this table are specified under $\gamma = 2.5$, $\tau = .025$ $S = 60$ *and* $v.0 = n+2$

The Markowitz model performs the worst of the models, both in terms of volatility and returns. A high volatility implies that the returns for each iteration are not consistent, which is a known feature of the Markowitz model. The results also imply that given the dataset, the historical mean and covariance do not do a great job on their own as data inputs in the portfolio optimization problem. This is consistent with the original hypothesis that further data inputs are necessary in conjunction with a more robust modeling procedure to improve the overall model.

The Black-Litterman model outperforms the Zhou model in both returns and volatility, meaning that in this analysis the incorporation of the historical data is not optimal. However, this does not render the Zhou model useless since repeated empirical analysis is necessary to determine the actual effects of the historical data. In Zhou only one iteration of the model is run as brief example, so there is currently no sufficient literature on whether the historical data is an optimal addition. A robust model testing procedure could be employed by running a rolling-window model testing procedure on many datasets, and then running t-tests on the set of returns and volatilities specified under each dataset to find if one model outperforms the other.

The inverse-Wishart prior performs significantly better than in volatility than all the other models, and is only beaten by the Black-Litterman model in returns. This is in line with the hypothesis that the inverse-Wishart prior will better specify the covariance matrix which will in turn lead to safer investment positions. Low volatility portfolios generally do not have high returns, and given that the volatility of the extension is

so much lower than the Black-Litterman volatility, it is not surprising that the return is also lower.

VII. Discussion

In exploring the results of the extended Zhou model it is clear that fully Bayesian models are able to outperform models that use loosely Bayesian methods. The inverse-Wishart extension outperforms the Zhou model in portfolio volatility by accounting for the uncertainty of modeling Σ and by allowing the investor to further specify confidence in the Black-Litterman and historical estimates. The parameters are straightforward and determined by the investor's confidence in each data input, which makes the model relatively simple and usable by any type of investor.

The Black-Litterman model, which is used as a joint prior in extended model, allows the investor to incorporate any sort of views on the market. The views can be determined in a one-off nature views or by a complex iterative function specifying a specific investment strategy. The former would likely be employed by an amateur, independent investor while the latter by a professional or investment team. The data updating stage has similar flexibility in that the historical means, or a more complex data modeling mechanism, can be employed depending on the quantitative skills of the investor. The incorporation of a predictive model is a topic of further research that could significantly increase the profitability of the Bayesian model, though it would also greatly increase the complexity. Asset return predictions models can also be incorporated in a much simpler manner through the use of absolute views.

The inverse-Wishart prior is used to model the uncertainty of predicting the next period's covariance matrix, which is not fully accounted for in the original Zhou model. This method works well empirically in this analysis, but further empirical testing is necessary to see if it consistently out-performs the Zhou model.

A further extension that could account for the problems in modeling Σ is through use a different estimate of Σ in the

equilibrium stage, rather than just the historical covariance. When many assets are being analyzed, the historical covariance matrix does estimate Σ well, so using another method of prediction could be very useful. Factor and stochastic volatility models could both provide another robust estimate of Σ in the equilibrium stage.

Another possible extension that is possible under the inverse-Wishart prior is to fully model the posterior predictive distribution, rather than simply using the mean value of the posterior inverse-Wishart distribution as the posterior estimate. The posterior predictive distribution under the inverse-Wishart prior is t-distributed, which may also be useful the since financial data is known to have fatter tails than the normal distribution. This would greatly increase the complexity of the model, however, since the expected utility would need to be maximized with respect to the posterior t-distribution, and this can only be done through complex integration.

The results presented in this paper give an idea of how the models perform under repeated conditions through the use of the rolling window. However, each iteration of the rolling window is very similar to the previous one since all but one data point is identical. In order to confidently determine if one model outperforms another, it is necessary to do an empirical analysis on multiple datasets.

As exemplified above, an investor can use many different strategies to specify the views, expected returns, and expected covariance matrix incorporated in the model. The method of combining these estimates is also quite important as seen by the optimal performance of the extended model, which used the same data inputs but incorporated an inverse-Wishart prior. By using Bayesian strategies to combine these different methods of prediction with the market equilibrium returns, the investor has a straightforward quantitative model that can help improve investment success. Almost all investors base their decisions off how they view the assets in the market, and by using this model, or variations of it, they can greatly improve their chance of profitability by using robust methods of

prediction.

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Multiproduct Pricing and Product Line Decisions with Status Externalities

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Abstract

In the present paper, reputation is approached as an idea involving status. We consider a multiproduct monopolist's product line and pricing decisions under the explicit assumption of two status externalities. The firm sells a low-end product and a high-end product to two segmented consumer groups. Whilst the sales of the high-end product increase the demand for the low-end product, the sales of the low-end product decrease the demand for the high-end product. If the products are not jointly branded, the status externalities do not exist. By performing comparative statics using the implicit function theorem we find that, given our assumptions, jointly branding products that were previously branded separately is associated with a high-end product price decrease and a low-end product price increase.

I. Introduction

Socrates is credited with saying that a good reputation is "the richest jewel you can possibly be possessed of." The importance of reputation is underlined in the economics literature, which focuses on two approaches to reputation (Cabral, 2005). The first approach, with hidden action, models repeated interaction and typically features moral hazard. It explores situations where a particular agent is expected to do something, such as breach a price fixing agreement. The second approach, with hidden information, models situations where a particular agent is thought to be something, such as a producer of high quality products, and typically features adverse selection (Cabral, 2005).

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In contrast to the above two strategies, we approach reputation as an idea involving status. Specifically, we explore the reputation of luxury brands and consider the effects of status externalities on the product line and pricing decisions of a monopolist selling status goods. There exists a demand for status goods, because for some it is desirable to be associated with wealth. According to Young, Nunes, and Dréze (2010), Thorsten Veblen argues in 'The Theory of the Leisure Class' (1899) that status is not exposed through the accumulation of wealth, but through its wasteful exhibition, described as "conspicuous consumption."

In order for conspicuous consumption to be effective, branding is utilized so that other consumers are able to recognize the status of the product. Young, Nunes and Drèze define brand prominence as "the extent to which a product has visible markings that help ensure observers recognize the brand." Products are attributed to having conspicuous or discrete branding. The relative conspicuousness of the branding attracts different types of customers and reflects the different signaling intentions of the consumer (Young, Nunes, and Drèze, 2010).

Young, Nunes and Drèze (2010) propose a taxonomy which assigns consumers into four groups, based on wealth and need for status. The set consists of patricians, parvenus, poseurs and proletarians. Patricians have wealth and a low need for status. They want to be associated with other patricians and pay a premium for inconspicuously branded products that use signals interpretable by only other patricians. Parvenus have wealth, and due to their need for high status they engage in conspicuous signaling. Their main concern is to be dissociated from the have-nots, whilst being associated with the wealthy. A luxury brand must make parvenus believe that proletarians and poseurs will know the brand and will recognize its consumer as wealthy. Poseurs have no wealth, but high need for status. They want to be associated with those recognizably wealthy, the parvenus, and dissociated from the have-nots. However, they cannot afford authentic luxury goods and therefore purchase counterfeit luxury products that act as inexpen-

sive substitutes. The proletarians have no wealth and no need for status. They neither want to associate nor dissociate with any of the other groups (Young, Nunes, and Drèze, 2010). By varying the price and conspicuousness of their brand, luxury goods manufacturers can target different types of consumers.

The demand for consumer goods and services can be broken down into functional and nonfunctional demand. Whilst functional demand is the part of the demand for a commodity which is due to the qualities inherent in the commodity itself, nonfunctional demand is that portion of the demand for a commodity which is due to factors other than the qualities inherent in the commodity (Leibenstein, 1950).

We categorize two types of nonfunctional demand. The first type is concerned with the Veblen effect, a phenomenon of conspicuous consumption, where there exists a willingness to pay a higher price for a functionally equivalent good with the intent to signal status (Bagwell and Bernheim, 1996). Whilst the Veblen effect is a function of price, the second type of nonfunctional demand is concerned with the "bandwagon" and "snob" effects, in which the utility derived from the commodity is respectively enhanced or decreased due to others consuming that commodity (Leibenstein, 1950).

The bandwagon effect causes an individual's demand for a commodity to increase when consumers in general or a specific group of individuals in the market demand more of the commodity. It represents the desire of people to purchase a commodity in order to be fashionable and to imitate people they want to be associated with. The snob effect is the reverse, in that the individual consumer's demand is negatively correlated with the total market demand. This represents the desire of people to be exclusive (Leibenstein, 1950). These effects have in common that the consumption behavior of any individual is not independent of the consumption of others.

Similar effects are described in the public policy literature. Positional goods are goods for which consumers are primarily concerned about relative consumption, and they are contrasted with nonpositional goods for which the consumer's main con-

cern is his or her absolute consumption. Frank (2005) suggests that economic models in which individual utility depends only on absolute consumption imply optimal allocation of positional and nonpositional goods. He argues, however, that economic models in which individual utility depends not only on absolute consumption, but also on relative consumption, predict in equilibrium too much expenditure on positional goods and too little on nonpositional goods, due to the expenditure arms races focused on positional goods (Frank, 2005).

This paper analyses a situation where demand is a function of the consumption of others. This leads to the idea of "non-additivity", as discussed in Frank (2005), which occurs when the market demand curve is not the lateral summation of the individual demand curves. We explore the product line and pricing decisions under the assumption of two status externalities. Specifically, the firm sells a low-end product and a high-end product to two segmented consumer groups and must decide whether to brand the products jointly or separately. If the firm sells the products under the same brand, sales of the high-end product positively affect the demand for the low-end product, whereas the sales of the low-end product negatively affect the demand for the high-end product. These effects arise due to the existence of status externalities, which arise under the assumption that sales of different products under the same brand have heterogeneous effects on the status reputation of the brand. Whilst a luxury brand may have high short-term sales by establishing a lower price line, this will diminish the exclusiveness associated with the brand. The following two examples explore benefits and costs of introducing low-end products.

Through the simultaneous introduction of the iPhone 5S and 5C, which occurred on the same day, Apple signaled a link between the two products. This link is further strengthened by the numerical component of the product names. However, as is apparent by the alphabetic component of their names, the two products are also differentiated. The more expensive 5S has more features, such as a fingerprint scanner. However,

from a branding perspective the most important difference in the products is their appearance. Apple's decision to utilize two separate materials and color schemes makes them distinguishable to the consumers. The 5S has an aluminum case and is available in the colors gray, silver and gold, which are traditionally associated with luxury. The less expensive 5C is built with a plastic case and is available in the colors blue, green, pink, yellow and white.

Apple is hoping to use the 5C to address emerging markets, especially China, where the majority of smartphone growth is projected to be in the low-end market. Apple entered a multi-year agreement with China Mobile, the world's largest mobile services provider by network scale and subscriber base, which serves over 760 million customers. As a result, the 5S and the 5C will be available in China Mobile and Apple retail stores across mainland China beginning in early 2014. This illustrates the increasing relevance of the Chinese market, which Apple CEO Tim Cook has described to be "extremely important" (Apple, Inc., 2013).

However, by moving towards the low-end market, Apple will possibly encounter difficulties. On the one hand there may be challenges capturing the Chinese low-end market because the 5C's price of $739 is relatively high compared with smartphones currently available in China, such as those from Huawei, Coolpad and ZTE, which are offered for less than $100 (Pfanner and Chen, 2013). On the other hand, since the 5C is relatively inexpensive compared to Apple's other products, its introduction may deter customers who value Apple's high status.

The product line of the luxury vehicle manufacturer Mercedes-Benz offers a further example of a traditional high-end brand targeting lower markets. In 2012, Mercedes-Benz began marketing the CLA-Class, which has a starting price of $29,900 and is the brand's lowest priced model to have entered the United States automobile retail market. The CLA-Class is expected to be Mercedes' bestselling model and has been called the company's current "most important car". In addition to

the expected increased revenue from the low-end market, this market's current clients will potentially become loyal to the brand and purchase higher priced models in the future. However, Mercedes-Benz must weigh the risks involved in targeting a more affordable market, since it may potentially deteriorate their high-end reputation (Stock, 2013).

Externalities associated with status products are not limited to the products of a single firm, but expand to the realm of two firms selling products that exhibit links to each other. In this case the status externalities could arise due to the product distribution's geographic proximity or close similarity in the product's design, which result in a strong association between the products. However, in these cases the status externalities are not internalized. An example is the market for counterfeits, where the two groups are the authentic producers and the counterfeiters.

Qian (2011) uses 1993-2004 product-line-level panel data on Chinese shoe companies to study the heterogeneous effects of counterfeit entry on sales of authentic products. The net effect of counterfeits on authentic product sales depends on the interplay of the negative substitution effects for authentic products and the positive advertising effects for a brand. The advertising effects arise when counterfeits enhance brand awareness and generate publicity for the brand, which signals brand popularity (Qian, 2011).

Qian (2011) finds that the advertising effect outweighs the substitution effect for the sales of high-end authentic products, which are less of a substitute for counterfeits. On the other hand, she finds that the substitution effect outweighs the advertising effect for low-end product sales. The net effect can vary even within the same brand. The net effect also differs between usage types. The positive advertising effect is most pronounced for high-fashion products, products tailored to a younger clientele, and products of younger brands that are less established at the time of the infringement (Qian 2011).

Qian (2011) gives recommendations for policy and business based on the heterogeneous impacts of counterfeiting. Coun-

terfeiting incentivizes authentic brands to upgrade their quality and signal higher quality to ensure that consumers can differentiate the authentic product from the counterfeits. The focus of intellectual property rights enforcement should be directed toward counterfeits that are substitutes for authentic products. In addition, relatively fewer enforcement resources can be devoted to products that benefit from the positive advertising effect (Qian, 2011).

Qian (2011) states that her findings, that the entry of counterfeits has both a negative substitution effect and a positive advertising effect, can be applied beyond the realm of counterfeiting. Since in this paper we assume the markets are completely segmented, the substitution effect does not apply to our model. In addition, applying the positive advertising effect to product lines suggests that sales of a low-end product positively affect the demand for the high-end product. However, in this paper we assume a negative reputation effect on the status of the established brand and that sales of a low-end product negatively affect the demand for the high-end product. In addition, in our analysis the externalities are internalized.

In this paper we consider the product line and pricing decisions of a multiproduct monopolist under the assumption of status externalities. We compare the case in which the firm sells the two products with different brands to the case in which the firm sells the two products under the same brand. If the products do not share a common brand, then there are no status externalities. We perform comparative statics using the implicit function theorem to study the characteristics of the market. We examine how prices change due to changes in the externalities. We find that jointly branding products that were previously sold with different brands is associated with a decrease in the price for the high-end product and an increase in the price for the low-end product.

II. The Model

An additively separable model is used to explore a multiproduct monopolist's product line and pricing decisions of two dif-

ferentiated status products, under the explicit assumption of two externalities. Specifically, whilst the sales of product 1 positively affect the demand for product 2, the sales of product 2 negatively affect the demand for product 1. The markets are completely segmented, due to which there is not spillage.

We implicitly assume product 1 is of higher quality and has higher status than product 2. We implicitly assume that the status externalities arise because the brand's status is associated with the average wealth of the brand's consumers. All consumers have preference to purchase from a brand whose clientele consists of wealthy consumers. The consumers of product 1 are wealthy, whereas the consumers of product 2 are not wealthy. Therefore, an increase in purchases by consumer 1 from the brand increases the average wealth of the brand's consumers and demand for product 2 increases. However, an increase in purchases by consumer 2 from the brand decreases the average wealth of the brand's consumers and demand for product 1 decreases.

We also implicitly assume that the two externalities are linked to network externalities. The bandwagon effect causes the demand of consumer 2 for product 2 to increase when consumer 1 purchases more of product 1, because it is desirable for consumer 2 to be fashionable and to purchase a product from the status brand from which wealthy consumers purchase products. However, due to the snob effect the demand of consumer 1 for product 1 is negatively related to sales of product 2, because consumer 1 values exclusivity.

The demand functions for product 1 q_1 and product 2 q_2 are linear combinations of the component of demand that is a function only of the own price of that product, respectively $D_1^*(p_1)$ and $D_2^*(p_2)$, and the component of demand given by the externality of the other good, respectively $-\beta_1 q_2$ and $\beta_2 q_1$, a change in which implies a shift in respectively q_1 and q_2. By the law of demand in both cases there is a negative relationship between the own price and quantity demanded. We assume $D_1^{*'}(p_1)$, $D_2^{*'}(p_2) < 0$.

$$q_1 = D_1(p_1, q_2) = D_1^*(p_1) - \beta_1 q_2$$
$$q_2 = D_2(p_2, q_1) = D_2^*(p_2) + \beta_2 q_1$$

We also assume that $D_1^*(p_1)$ and $D_2^*(p_2)$ are twice continuously differentiable and that $D_1^{*''}(p_1)$,$D_2^{*''}(p_2) < 0$. That is, we assume the demand functions for product 1 and product 2 to be strictly concave (Mas-Colell, Whinston, and Green, 1995).

A demand function being strictly decreasing and strictly concave implies that for a given price change in absolute terms, the price change is associated with a larger decrease in the quantity demanded if the price change occurs at a higher price than if it were to occur at a lower price.

We think this is a reasonable assumption in the market for luxury goods. Starting at a low price, a price increase may initially be associated with only a small decrease in the quantity demanded due to customer loyalty and because other luxury products may not be perfect substitutes. Starting at a low price, a price decrease may be associated with only a small increase in the quantity demanded since consumers' demand may already be saturated (Scott, 1997).

However, starting at a high price a price increase may be associated with a large decrease in the quantity demanded, since consumers are increasingly deterred from purchasing the good. Consumers may choose to purchase a similar luxury good from a substitute luxury brand or choose not to purchase a luxury good at all since it is not considered a necessity. We rewrite the demand as functions of p_1 and p_2.

$$q_1 = D_1(p_1, p_2) = \frac{D_1^*(p_1) - \beta_1 D_2^*(p_2)}{(1 + \beta_1\beta_2)}$$
$$q_2 = D_2(p_1, p_2) = \frac{D_2^*(p_2) + \beta_2 D_1^*(p_1)}{(1 + \beta_1\beta_2)}$$

The magnitudes of the externalities are measured by β_1, β_2. To assure that q_1,$q_2 \geq 0$, we restrict $0 \leq \beta_1 \leq \frac{D_1^*(p_1)}{D_2^*(p_2)}$

and $0 \leq \beta_2$. Note that if $\beta_1 = 0$, then $q_1 = D_1(p_1, q_2) = D_1^*(p_1)$. Similarly, if $\beta_2 = 0$, then $q_2 = D_2(p_2, q_1) = D_2^*(p_2)$. We implicitly assume that not branding the products together implies that β_1=β_2=0.

Given the above assumptions the price effects are:

$$\begin{bmatrix} \frac{\partial q_1}{\partial p_1} & \frac{\partial q_1}{\partial p_2} \\ \frac{\partial q_2}{\partial p_1} & \frac{\partial q_2}{\partial p_2} \end{bmatrix} = \begin{bmatrix} \frac{D_1^{*\prime}(p_1)}{(1+\beta_1\beta_2)} & \frac{-\beta_1 D_2^{*\prime}(p_2)}{(1+\beta_1\beta_2)} \\ \frac{\beta_2 D_1^{*\prime}(p_1)}{(1+\beta_1\beta_2)} & \frac{D_2^{*\prime}(p_2)}{(1+\beta_1\beta_2)} \end{bmatrix} = \begin{bmatrix} - & + \\ - & - \end{bmatrix}$$

The relationship between $p_2$and q_1 is that which we would expect in the case of substitutes, whereas the relationship between p_1 and q_2 is that which we would expect in the case of complements.

Let R_1 and R_2 be the respective revenues generated through sales of products 1 and 2.

$$R_1 = p_1 q_1 = p_1 \frac{D_1^*(p_1) - \beta_1 D_2^*(p_2)}{(1+\beta_1\beta_2)}$$

$$R_2 = p_2 q_2 = p_2 \frac{D_2^*(p_2) + \beta_2 D_1^*(p_1)}{(1+\beta_1\beta_2)}$$

Let the total revenue be R=$R_1 + R_2$

$$R = p_1 \frac{D_1^*(p_1) - \beta_1 D_2^*(p_2)}{(1+\beta_1\beta_2)} + p_2 \frac{D_2^*(p_2) + \beta_2 D_1^*(p_1)}{(1+\beta_1\beta_2)}$$

Let the total cost be given by $C(q_1, q_2) = F + c_1 q_1 + c_2 q_2$, where F is a constant shared fixed cost, and c_1, c_2 are constant per unit costs, such that $c_1 > c_2$, since product 1 is the status good, which is of a higher quality that product 2. We rewrite the total cost C as a function of p_1, p_2.

$$C = F + c_1 \frac{D_1^*(p_1) - \beta_1 D_2^*(p_2)}{(1 + \beta_1\beta_2)} + c_2 \frac{D_2^*(p_2) + \beta_2 D_1^*(p_1)}{(1 + \beta_1\beta_2)}$$

Let profit π be a real-valued function of p_1, p_2. Let $\pi : A \subset R^2 \to R$, where $A = \{(p_1, p_2) \in R^2 \mid 0 < p_1, p_2 < \infty\}$.

$$\pi(p_1, p_2) = (p_1 - c_1)\frac{D_1^*(p_1) - \beta_1 D_2^*(p_2)}{(1 + \beta_1\beta_2)} +$$

$$(p_2 - c_2)\frac{D_2^*(p_2) + \beta_2 D_1^*(p_1)}{(1 + \beta_1\beta_2)} - F$$

We want to determine the maxima of π. A point $p_0 \in A$ is a critical point if π is differentiable at p_0 and if $D\pi(p_0) = 0$ (Marsden and Hoffman, 1993). We take the partial derivatives of π with respect to of p_1 and p_2 and set the partial derivatives equal to zero.

$$\frac{\partial \pi}{\partial p_1} = \frac{D_1^*(p_1) - \beta_1 D_2^*(p_2) + (p_1 - c_1)D_1^{*\prime}(p_1) + (p_2 - c_2)\beta_2 D_1^{*\prime}(p_1)}{(1 + \beta_1\beta_2)}$$

$$\frac{\partial \pi}{\partial p_2} = \frac{(p_1 - c_1)(-\beta_1)D_2^{*\prime}(p_2) + D_2^*(p_2) + \beta_2 D_1^*(p_1) + (p_2 - c_2)D_2^{*\prime}(p_2)}{(1 + \beta_1\beta_2)}$$

If p_0 is an extreme point, either a local minimum or a local maximum for π, then p_0 is a critical point. However, if p_0 is a critical point, it does not necessarily imply that p_0 is an extreme point (Marsden and Hoffman, 1993). Furthermore, even if p_0 is an extreme point, we want to test that at this point π is maximized. We check whether the second order condition is satisfied, that is the Hessian matrix of second derivatives is negative definite. The Hessian matrix is:

$$\begin{bmatrix} \frac{\partial^2 \pi}{\partial {p_1}^2} & \frac{\partial^2 \pi}{\partial p_1 \partial p_2} \\ \frac{\partial^2 \pi}{\partial p_2 \partial p_1} & \frac{\partial^2 \pi}{\partial {p_2}^2} \end{bmatrix}$$

We evaluate the partial derivatives at $p_0 = (p_1^m, p_2^m)$:

$$\begin{bmatrix} \frac{2D_1^{*\prime}(p_1^m) + D_1^{*\prime\prime}(p_1^m)((p_1^m - c_1) + \beta_2(p_2^m - c_2))}{(1+\beta_1\beta_2)} & \frac{\beta_2 D_1^{*\prime}(p_1^m) - \beta_1 D_2^{*\prime}(p_2^m)}{(1+\beta_1\beta_2)} \\ \frac{\beta_2 D_1^{*\prime}(p_1^m) - \beta_1 D_2^{*\prime}(p_2^m)}{(1+\beta_1\beta_2)} & \frac{2D_2^{*\prime}(p_2^m) + D_2^{*\prime\prime}(p_2^m)\big((p_2^m - c_2) - \beta_1(p_1^m - c_1)\big)}{(1+\beta_1\beta_2)} \end{bmatrix}$$

First we note that both diagonal elements are negative. This is assured through our assumption that

$$D_1^{*\prime}(p_1), D_2^{*\prime}(p_2), D_1^{*\prime\prime}(p_1), D_2^{*\prime\prime}(p_2) < 0$$

Second, we want to check that the determinant of the matrix is positive. The determinant is:

$$\frac{\Big(2D_1^{*\prime}(p_1^m) + D_1^{*\prime\prime}(p_1^m)\big((p_1^m - c_1) + \beta_2(p_2^m - c_2)\big)\Big)\Big(2D_2^{*\prime}(p_2^m) + D_2^{*\prime\prime}(p_2^m)\big((p_2^m - c_2) - \beta_1(p_1^m - c_1)\big)\Big)}{(1+\beta_1\beta_2)^2} - \frac{\Big(\beta_2 D_1^{*\prime}(p_1^m) - \beta_1 D_2^{*\prime}(p_2^m)\Big)^2}{(1+\beta_1\beta_2)^2}$$

We cannot sign the determinant in general. However, evaluated at $\beta_1 = \beta_2 = 0$, the determinant is positive:

$$\Big(2D_1^{*\prime}(p_1^m) + D_1^{*\prime\prime}(p_1^m)(p_1^m - c_1)\Big)\Big(2D_2^{*\prime}(p_2^m) + D_2^{*\prime\prime}(p_2^m)(p_2^m - c_2)\Big) > 0$$

We assume that at p_0 π is maximized in general. We denote the multiproduct optimum. The multiproduct monopolist chooses the multiproduct monopoly prices p_1^m and p_2^m which determine the multiproduct monopoly outputs q_1^m and q_2^m. The two equations below implicitly define the multiproduct monopoly prices p_1^m and p_2^m as functions of β_1 and β_2.

$$\frac{D_1^*(p_1^m) - \beta_1 D_2^*(p_2^m) + D_1^{*\prime}(p_1^m)((p_1^m - c_1) + \beta_2(p_2^m - c_2))}{(1 + \beta_1\beta_2)} = 0$$

$$\frac{D_2^*(p_2^m) + \beta_2 D_1^*(p_1^m) + D_2^{*\prime}(p_2^m)((p_2^m - c_2) - \beta_1(p_1^m - c_1))}{(1 + \beta_1\beta_2)} = 0$$

We will now calculate the single product monopoly optimum. If the firm operates only in the high-end market $q_2 = 0$ and $q_1 = D_1^*(p_1)$. The revenue generated through sales of product 1 is $R_1 = p_1 q_1 = p_1 D_1^*(p_1)$. Given that F_1 is the constant fixed cost, and c_1 is the constant per unit, the total cost in terms of p_1 is $C = F + c_1 D_1^*(p_1)$.

Let profit π be a real-valued function of p_1. Let $\pi : A \subset R \to R$, where $A = \{p_1 \in R \mid 0 < p_1 < \infty\}$, $\pi(p_1) = (p_1 - c_1)D_1^*(p_1) - F$. We want to determine the maxima of π. A point $p_0 \in A$ is a critical point if π is differentiable at p_0 and if $D\pi(p_0) = 0$ (Marsden and Hoffman, 1993). We take the partial derivative of π with respect to p_1 and set it equal to zero.

$$\frac{\partial \pi}{\partial p_1} = D_1^*(p_1) + (p_1 - c_1)D_1^{*\prime}(p_1)$$

The second order condition is satisfied, because the second derivative is negative.

$$\frac{\partial^2 \pi}{\partial {p_1}^2} = 2D_1^{*\prime}(p_1) + D_1^{*\prime\prime}(p_1)(p_1 - c_1) < 0$$

Therefore, the single product monopolist chooses the single product monopoly price p_1^s, which determines the single product monopoly output q_1^s.

$$D_1^*(p_1^s) + (p_1^s - c_1)D_1^{*\prime}(p_1^s) = 0$$

Similarly, if the firm operates only in the low-end market, then $q_1 = 0$ and $q_2 = D_2^*(p_2)$. In this case, the single product monopolist chooses the single product monopoly price p_2^s which determine the single product monopoly output q_2^s.

$$D_2^*(p_2^s) + (p_2^s - c_2)D_1^{*\prime}(p_2^s) = 0$$

The single product monopoly pricing for a firm which operates either only in the high-end market or only in the low-end market, which results in zero sales of the other product, is equivalent to the multiproduct monopolist optimum when the demands are independent, $\beta_1 = \beta_2 = 0$. There are no externalities if the products do not share a common brand. If the multiproduct monopolist separately brands the two products, the respective demands are:

$$q_1 = D_1^*(p_1)$$
$$q_2 = D_2^*(p_2)$$

The first order conditions are:

$$D_1^*(p_1^s) + (p_1^s - c_1)D_1^{*\prime}(p_1^s) = 0$$
$$D_2^*(p_2^s) + (p_2^s - c_2)D_1^{*\prime}(p_2^s) = 0$$

We define $\epsilon_{11}^s = \frac{\partial q_1}{\partial p_1}\frac{p_1}{q_1}$ and $\epsilon_{22}^s = \frac{\partial q_2}{\partial p_2}\frac{p_2}{q_2}$ to be the respective price elasticity of demand for the case where the products are branded separately. We rewrite the first order conditions:

$$\frac{(p_1^s - c_1)}{p_1^s} = -\frac{1}{\varepsilon_{11}^s}$$
$$\frac{(p_2^s - c_2)}{p_2^s} = -\frac{1}{\varepsilon_{22}^s}$$

In the case where externalities exist, the two equations below implicitly define the multiproduct monopoly prices p_1^m and p_2^m as functions of β_1 and β_2.

$$\frac{D_1^*(p_1^m) - \beta_1 D_2^*(p_2^m) + D_1^{*\prime}(p_1^m)((p_1^m - c_1) + \beta_2(p_2^m - c_2))}{(1 + \beta_1\beta_2)} = 0$$

$$\frac{D_2^*(p_2^m) + \beta_2 D_1^*(p_1^m) + D_2^{*\prime}(p_2^m)((p_2^m - c_2) - \beta_1(p_1^m - c_1))}{(1 + \beta_1\beta_2)} = 0$$

The price elasticity of demand is:

$$\varepsilon_{11}^m = \frac{\partial q_1}{\partial p_1}\frac{p_1}{q_1} = \frac{p_1 D_1^{*\prime}(p_1)}{D_1^*(p_1) - \beta_1 D_2^*(p_2)} < 0$$

$$\varepsilon_{22}^m = \frac{\partial q_2}{\partial p_2}\frac{p_2}{q_2} = \frac{p_2 D_2^{*\prime}(p_2)}{D_2^*(p_2) + \beta_2 D_1^*(p_1)} < 0$$

The cross-price elasticity of demand is:

$$\varepsilon_{12}^m = \frac{\partial q_1}{\partial p_2}\frac{p_2}{q_1} = \frac{-p_2\beta_1 D_2^{*\prime}(p_2)}{D_1^*(p_1) - \beta_1 D_2^*(p_2)} > 0$$

$$\varepsilon_{21}^m = \frac{\partial q_2}{\partial p_1}\frac{p_1}{q_2} = \frac{p_1\beta_2 D_1^{*\prime}(p_1)}{D_2^*(p_2) + \beta_2 D_1^*(p_1)} < 0$$

As for substitutes ϵ_{12}^m is positive and as for complements ϵ_{21}^m is negative. The cross-price elasticity of demand is zero if the demands are independent, which is the case if $\beta_1 = \beta_2 = 0$.

We rewrite the first order conditions for the case where the products are jointly branded and solve for price:

$$p_1^m = \frac{c_1 - \beta_2(p_2^m - c_2)}{\left(1 + \frac{1}{\varepsilon_{11}^m}\right)}$$

$$p_2^m = \frac{c_2 + \beta_1(p_1^m - c_1)}{\left(1 + \frac{1}{\varepsilon_{22}^m}\right)}$$

Since a monopolist does not produce on the inelastic portion of the demand curve, ϵ_{11}^m,$\epsilon_{22}^m < -1$ (Pindyck and Ru-

binfeld, 2009). Therefore, all else equal an increase in β_2 is associated with a decrease in p_1^m, whereas all else equal an increase in β_1 is associated with an increase in p_2^m. This analysis is possible since ϵ_{11}^m is independent of β_2 and ϵ_{22}^m is independent of β_1.

III. The Effect of Changes in Externalities on Pricing

This section examines how prices change due to changes in the magnitude of the spillover parameters. However, we first explore possible reasons for the existence of and changes in the externalities. We implicitly assume that the externalities exist due to a link between the low-end and the high-end products, as well as a link between the two groups of consumers. Due to the links consumers associate the low-end product with the high-end product. Therefore, a change in the sales for the one product will result in a change in the demand of the other product. An increase in the links is associated with an increase in the size of the externalities. The links are based on the interaction of the two consumer groups during which the product, which jointly branded with the product that can the purchased by the other consumer group, is displayed. Marketing can be used to alter the intensity of the link.

We assume implicitly that the link between the two groups of customers is affected by the extent to which the two groups of customers have information about each other's purchases. This depends on how frequently the two groups interact, since through interaction the purchase decisions are exhibited. The interaction is not limited to direct in-person interaction, but can occur indirectly via various media. The frequency of the interaction may be limited due to geographic or social barriers.

In addition to the frequency of the interactions between the two groups, we consider the extent to which the products are displayed throughout these interactions. Therefore, an additional factor that has an effect on the magnitude of the externalities is whether the products are consumed privately or publically. Products that are frequently displayed in public,

such as accessories or smartphones, are associated with larger status externalities in absolute value than products that are usually consumed in private, such as furniture.

Furthermore, a link is established between the low-end and high-end products if the firm sells the two products under the same brand. If the products have recognizable similar features, such as name, logo, and design, the products will have a shared identity and therefore influence each other's reputation. In the case of status externalities, if the brand is associated with exclusiveness, a lower priced product may diminish the exclusivity and possibly decrease the value of the brand, since high-end exclusive customers want to dissociate themselves from the other group. If the firm brands the products separately, we assume that the link does not exist, $\beta_1 = \beta_2 = 0$, and neither the positive spillover $\beta_2\ q_1$ nor the negative spillover $\beta_1\ q_2$ would be experienced. The firm's decision to brand the products jointly or separately depends on the relative importance of the markets. The firm could also have sub-brands or luxury and regular product lines.

Furthermore, marketing can communicate to consumers the extent to which the products are similar or dissimilar, and therefore alter the link. The firm can, for example, advertise the exclusiveness of the high-end product. The firm can also advertise the products together and underline their similarities. In addition, high volumes of advertisement and prominent branding are associated with larger externalities because this results in the brand being more known and recognizable by the public. The firm may consider utilizing advertisement to minimize the magnitude of the negative externality β_1 and maximize the magnitude of the positive externality β_2.

We perform comparative statics using the implicit function theorem. The equilibrium prices p_1 and p_2 are implicitly defined as functions of β_1 and β_2 in the two equations that are yielded through the first order conditions.

$$F_1(p_1,p_2,\beta_1,\beta_2) = \frac{D_1^*(p_1) - \beta_1 D_2^*(p_2) + D_1^{*\prime}(p_1)((p_1 - c_1) + \beta_2(p_2 - c_2))}{(1+\beta_1\beta_2)}$$

$$F_2(p_1,p_2,\beta_1,\beta_2) = \frac{D_2^*(p_2) + \beta_2 D_1^*(p_1) + D_2^{*\prime}(p_2)((p_2 - c_2) - \beta_1(p_1 - c_1))}{(1+\beta_1\beta_2)}$$

$$F_1(p_1,p_2,\beta_1,\beta_2) = 0$$

$$F_2(p_1,p_2,\beta_1,\beta_2) = 0$$

We restrict F_1 and F_2 to the level set where F_1,F_2=0, values of p_1,p_2,β_1,and β_2 such that F_1,F_2=0, and denote the restriction F_1^r and F_2^r. The partial derivatives of F_1^r and F_2^r with respect to p_1,p_2,β_1,and β_2, are equal to the partial derivatives of F_1 and F_2 with respect p_1,p_2,β_1,and β_2, however, the manipulation is simplified.

$$F_1^r(p_1,p_2,\beta_1,\beta_2) = D_1^*(p_1) - \beta_1 D_2^*(p_2) + D_1^{*\prime}(p_1)((p_1 - c_1) + \beta_2(p_2 - c_2))$$

$$F_2^r(p_1,p_2,\beta_1,\beta_2) = D_2^*(p_2) + \beta_2 D_1^*(p_1) + D_2^{*\prime}(p_2)((p_2 - c_2) - \beta_1(p_1 - c_1))$$

We want to determine the sign of each component of the matrix of partial derivatives of price with respect to β_1,β_2. To do so we calculate an expression for each component.

$$\begin{bmatrix} \frac{\partial p_1}{\partial \beta_1} & \frac{\partial p_1}{\partial \beta_2} \\ \frac{\partial p_2}{\partial \beta_1} & \frac{\partial p_2}{\partial \beta_2} \end{bmatrix} = -\begin{bmatrix} \frac{\partial F_1^r}{\partial p_1} & \frac{\partial F_1^r}{\partial p_2} \\ \frac{\partial F_2^r}{\partial p_1} & \frac{\partial F_2^r}{\partial p_2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial F_1^r}{\partial \beta_1} & \frac{\partial F_1^r}{\partial \beta_2} \\ \frac{\partial F_2^r}{\partial \beta_1} & \frac{\partial F_2^r}{\partial \beta_2} \end{bmatrix}$$

$$= -\left(\frac{\partial F_1^r}{\partial p_1}\frac{\partial F_2^r}{\partial p_2} - \frac{\partial F_1^r}{\partial p_2}\frac{\partial F_2^r}{\partial p_1}\right)^{-1} \begin{bmatrix} \frac{\partial F_2^r}{\partial p_2}\frac{\partial F_1^r}{\partial \beta_1} - \frac{\partial F_1^r}{\partial p_2}\frac{\partial F_2^r}{\partial \beta_1} & \frac{\partial F_2^r}{\partial p_2}\frac{\partial F_1^r}{\partial \beta_2} - \frac{\partial F_1^r}{\partial p_2}\frac{\partial F_2^r}{\partial \beta_2} \\ \frac{\partial F_1^r}{\partial p_1}\frac{\partial F_2^r}{\partial \beta_1} - \frac{\partial F_2^r}{\partial p_1}\frac{\partial F_1^r}{\partial \beta_1} & \frac{\partial F_1^r}{\partial p_1}\frac{\partial F_2^r}{\partial \beta_2} - \frac{\partial F_2^r}{\partial p_1}\frac{\partial F_1^r}{\partial \beta_2} \end{bmatrix}$$

Based on our current assumptions, the signs of two of the below partial derivatives cannot be determined.

$$\frac{\partial F_1^r}{\partial p_1} = 2D_1^{*\prime}(p_1) + D_1^{*\prime\prime}(p_1)\big((p_1 - c_1) + \beta_2(p_2 - c_2)\big) < 0$$

$$\frac{\partial F_1^r}{\partial p_2} = \beta_2 D_1^{*\prime}(p_1) - \beta_1 D_2^{*\prime}(p_2)\ ?$$

$$\frac{\partial F_1^r}{\partial \beta_1} = -D_2^*(p_2) < 0$$

$$\frac{\partial F_1^r}{\partial \beta_2} = D_1^{*\prime}(p_1)(p_2 - c_2) < 0$$

$$\frac{\partial F_2^r}{\partial p_1} = \beta_2 D_1^{*\prime}(p_1) - \beta_1 D_2^{*\prime}(p_2)\ ?$$

$$\frac{\partial F_2^r}{\partial p_2} = 2D_2^{*\prime}(p_2) + D_2^{*\prime\prime}(p_2)\big((p_2 - c_2) - \beta_1(p_1 - c_1)\big) < 0$$

$$\frac{\partial F_2^r}{\partial \beta_1} = -D_2^{*\prime}(p_2)(p_1 - c_1) > 0$$

$$\frac{\partial F_2^r}{\partial \beta_2} = D_1^*(p_1) > 0$$

In each of the entries of the matrix below the sign of one of the partial derivatives cannot be determined. The two partial derivatives whose sign cannot the determined are equal, $\frac{\partial F_1^r}{\partial p_2} = \frac{\partial F_2^r}{\partial p_1}$. Therefore, their product is either zero or positive. However, if their product is a nonzero positive we cannot sign the determinant.

$$\begin{bmatrix} \frac{\partial p_1}{\partial \beta_1} & \frac{\partial p_1}{\partial \beta_2} \\ \frac{\partial p_2}{\partial \beta_1} & \frac{\partial p_2}{\partial \beta_2} \end{bmatrix} = -\left(\frac{\partial F_1^r}{\partial p_1}\frac{\partial F_2^r}{\partial p_2} - \frac{\partial F_1^r}{\partial p_2}\frac{\partial F_2^r}{\partial p_1}\right)^{-1} \begin{bmatrix} \frac{\partial F_2^r}{\partial p_2}\frac{\partial F_1^r}{\partial \beta_1} - \frac{\partial F_1^r}{\partial p_2}\frac{\partial F_2^r}{\partial \beta_1} & \frac{\partial F_2^r}{\partial p_2}\frac{\partial F_1^r}{\partial \beta_2} - \frac{\partial F_1^r}{\partial p_2}\frac{\partial F_2^r}{\partial \beta_2} \\ \frac{\partial F_1^r}{\partial p_1}\frac{\partial F_2^r}{\partial \beta_1} - \frac{\partial F_2^r}{\partial p_1}\frac{\partial F_1^r}{\partial \beta_1} & \frac{\partial F_1^r}{\partial p_1}\frac{\partial F_2^r}{\partial \beta_2} - \frac{\partial F_2^r}{\partial p_1}\frac{\partial F_1^r}{\partial \beta_2} \end{bmatrix}$$

To build intuition we will examine to the case where β_1,$\beta_2 > 0$, but the derivatives are evaluated at $\beta_1 = 0$. Hence, we evaluate the impact of the externalities at the point where β_1 starts with no impact. Given this assumption, the signs of the other partial derivatives remain unchanged and we sign:

$$\frac{\partial F_1^r}{\partial p_2} = \frac{\partial F_2^r}{\partial p_1} = \beta_2 D_1^{*\prime}(p_1) < 0, \text{evaluated at } \beta_1 = 0$$

However, under this assumption, we are still unable to sign the determinant.

We now examine the case where $\beta_1, \beta_2 > 0$, but the derivatives are evaluated at $\beta_2 = 0$. Hence, we evaluate the impact of the externalities at the point where β_2 starts with no impact. Given this assumption the signs of the other partial derivatives remain unchanged and we sign:

$$\frac{\partial F_1^r}{\partial p_2} = \frac{\partial F_2^r}{\partial p_1} = -\beta_1 D_2^{*\prime}(p_2) > 0, \text{evaluated at } \beta_2 = 0$$

However, under this assumption, we are still unable to sign the determinant.

Given the previous two assumptions separately, we could not sign the determinant. Hence, we explore the case where, $\beta_1, \beta_2 > 0$, but the derivatives are evaluated at $\beta_1 = \beta_2 = 0$. Hence we evaluate the impact of the externalities at the point where both β_1 and β_2 start with no impact. Given these assumptions, the signs of the other partial derivatives remain unchanged and we sign:

$$\frac{\partial F_1^r}{\partial p_2} = \frac{\partial F_2^r}{\partial p_1} = 0, \text{evaluated at } \beta_1 = \beta_2 = 0$$

We derived earlier that:

$$\begin{bmatrix} \frac{\partial p_1}{\partial \beta_1} & \frac{\partial p_1}{\partial \beta_2} \\ \frac{\partial p_2}{\partial \beta_1} & \frac{\partial p_2}{\partial \beta_2} \end{bmatrix} = -\left(\frac{\partial F_1^r}{\partial p_1}\frac{\partial F_2^r}{\partial p_2} - \frac{\partial F_1^r}{\partial p_2}\frac{\partial F_2^r}{\partial p_1}\right)^{-1} \begin{bmatrix} \frac{\partial F_2^r}{\partial p_2}\frac{\partial F_1^r}{\partial \beta_1} - \frac{\partial F_1^r}{\partial p_2}\frac{\partial F_2^r}{\partial \beta_1} & \frac{\partial F_2^r}{\partial p_2}\frac{\partial F_1^r}{\partial \beta_2} - \frac{\partial F_1^r}{\partial p_2}\frac{\partial F_2^r}{\partial \beta_2} \\ \frac{\partial F_1^r}{\partial p_1}\frac{\partial F_2^r}{\partial \beta_1} - \frac{\partial F_2^r}{\partial p_1}\frac{\partial F_1^r}{\partial \beta_1} & \frac{\partial F_1^r}{\partial p_1}\frac{\partial F_2^r}{\partial \beta_2} - \frac{\partial F_2^r}{\partial p_1}\frac{\partial F_1^r}{\partial \beta_2} \end{bmatrix}$$

Given the assumption where the partial derivatives are evaluated at $\beta_1 = \beta_2 = 0$, we get:

$$\begin{bmatrix} \frac{\partial p_1}{\partial \beta_1} & \frac{\partial p_1}{\partial \beta_2} \\ \frac{\partial p_2}{\partial \beta_1} & \frac{\partial p_2}{\partial \beta_2} \end{bmatrix} = -\left(\frac{\partial F_1^r}{\partial p_1}\frac{\partial F_2^r}{\partial p_2}\right)^{-1} \begin{bmatrix} \frac{\partial F_2^r}{\partial p_2}\frac{\partial F_1^r}{\partial \beta_1} & \frac{\partial F_2^r}{\partial p_2}\frac{\partial F_1^r}{\partial \beta_2} \\ \frac{\partial F_1^r}{\partial p_1}\frac{\partial F_2^r}{\partial \beta_1} & \frac{\partial F_1^r}{\partial p_1}\frac{\partial F_2^r}{\partial \beta_2} \end{bmatrix} = \begin{bmatrix} - & - \\ + & + \end{bmatrix}$$

Therefore, $\frac{\partial p_1}{\partial \beta_1} < 0$, $\frac{\partial p_1}{\partial \beta_2} < 0$, $\frac{\partial p_2}{\partial \beta_1} > 0$, and $\frac{\partial p_2}{\partial \beta_2} > 0$. All else equal, an increase in either β_1 or β_2 is associated with a decrease in F_1^r and an increase in F_2^r. We have shown that in the neighborhood of $\beta_1 = \beta_2 = 0$, the restoration of equilibrium necessitates a decrease in p_1 and an increase in p_2. An increase in β_1 and β_2, starting at $\beta_1 = \beta_2 = 0$, is representative of moving from the case in which the firm sells two products with different brands to the case in which the firm sells two products under the same brand. Therefore, jointly branding products that were previously sold with different brands is associated with a decrease in the price for the high-end product and an increase in the price for the low-end product.

We first explain the intuition behind why an increase in β_1 implies, in equilibrium, a decrease in p_1 and an increase in p_2. All else equal, an increase in β_1 implies that the first market is hurt more by sales of product 2. Therefore, to restore equilibrium, an increase in β_1 is associated with an increase in p_2, since this leads to a decrease in $D_2^* p_2$ so that less of product 2 is sold, so that demand for product 1 remains high and the damage is dampened in market 1, at the loss of less profit in market 2. Whilst an increase in p_2 shifts up demand for market 1, the demand in market 1 is nevertheless lower than before the increase in β_1 and therefore, to restore an optimum, p_1 is decreased.

We explain the intuition behind why an increase in β_2 implies, in equilibrium, a decrease in p_1 and an increase in p_2. All else equal, an increase in β_2 implies that market 2 benefits more by sales of product 1. To restore equilibrium, an increase in β_2 is associated with a decrease in p_1, since this leads to an increase in $D_1^* p_1$ so that more of product 1 is sold, so that demand for product 2 is further increased and profit in market 2 is further increased, at the loss of less profit in market 1.

Given our model, an increase in demand for product 2 implies an increase in p_2.

IV. Discussion and Conclusions

This paper investigates a multiproduct monopolist's product line and pricing decisions of two differentiated status products, under the explicit assumption of two externalities. Specifically, whilst the sales of the high-end product positively affect the demand for the low-end product, the sales of the low-end product negatively affect the demand for the high-end product. We find that jointly branding products, which were previously sold with different brands, is associated with a decrease in the price for the high-end product and an increase in the price for the low-end product.

Whilst it necessitates empirical tests of the model to investigate its value of representing observable reality, we will outline ways in which the model and analysis can be improved and extended. First, the model could be improved by making the assumptions explicit and deriving the demand functions from assumptions on preferences. Demand could be derived as a function of the consumer's wealth, the quality of the product, and the status of the brand, which could be the average wealth of the consumer who purchases from the brand. In addition to making the status externalities explicit, an improvement would be not assuming that markets are completely segmented and allowing spillage, which allows a low-priced product to cannibalize the high-priced product. This would better depict reality, where some wealthy consumers purchase low-end products and some non-wealthy consumers purchase high-end products.

Modeling relative price differences is an improvement of the model that does not involve assumptions about quality. The intuition is that if p_1 increases the brand is associated with even more status, whereas if p_2 decreases the brand status is even further diminished. Below is a possible model, where if $p_1 = p_2$ the externalities do not exist.

$$q_1 = D_1(p_1, q_2) = D_1^*(p_1) - \beta_1 \left(\frac{p_1 - p_2}{p_2}\right) q_2$$

$$q_2 = D_2(p_2, q_1) = D_2^*(p_2) + \beta_2 \left(\frac{p_1 - p_2}{p_2}\right) q_1$$

Furthermore, additional externalities, such as an advertising effect (Qian, 2011) or network externalities that affect the products own demand, could make the model represent reality more accurately. The model could also be generalized to an array of products that vary in quality and an array of market segments that vary in size. A further consideration is to model competition, where firms react to each other's price changes. The interaction of costs could also be modeled. Further analysis might also consider maximization of total welfare.

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Notes

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